JAMAICA COLLEGE

C.S.E.C. MATHEMATICS MARATHON (PART ONE)

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FRACTIONS

A fraction is a part of a whole.
It is a measure of how the whole is divided or shared.

A fraction is usually written in the form $\frac{n}{d}$, where n and d are natural (counting numbers). Here, n is called the numerator and d is called the denominator.

Proper fractions

A proper fraction is a fraction where the numerator is LESS THAN the denominator.
In this case, the fraction is less than one (the whole).

Examples of proper fractions are $\frac{2}{3}, \frac{11}{15}, \frac{9}{29}$ and $\frac{37}{44}$.

Improper fractions

An improper fraction is a fraction where the numerator is MORE THAN the denominator.
In this case, the fraction is more than one (the whole).

Examples of improper fractions are $\frac{5}{2}, \frac{14}{9}, \frac{53}{19}$ and $\frac{7}{3}$.

Lowest common multiple (LCM)

The lowest common multiple (LCM) of two natural numbers, p and q, is the lowest natural number which is divisible by BOTH p and q.

Consider the numbers 4 and 6.

Multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36, 40 etc
Multiples of 6 are 6, 12, 18, 24, 30, 36, 42 etc

So, common multiples of 4 and 6 are 12, 24, 36 etc
The least of all common multiples of 4 and 6 is 12.
So, the least common multiple (LCM) of 4 and 6 is 12.

Equivalent fractions

If $\frac{n}{d}$ is fraction, then we may multiply or divide BOTH numerator and denominator by a natural (counting) number.

For example, $\frac{1}{2} = \frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$

Also, $\frac{1}{2} = \frac{1}{2} \times \frac{17}{17} = \frac{17}{34}$

So, $\frac{3}{6}$ and $\frac{17}{34}$ are equivalent fractions to $\frac{1}{2}$.

Highest common factor (HCF)

The highest common factor (HCF) of two natural numbers, p and q, is the highest natural number which can go into BOTH p and q.

Consider the numbers 15 and 24.

The factors of 15 are 1, 3, 5 are 15
The factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24

So, the common factors of 15 and 24 are 1 and 3.
The greatest of all common factors of 15 and 24 is 3.
So, the highest common factor (HCF) of 15 and 24 is 3.

Simplifying fractions

$$\frac{48}{72} = \frac{3\times 16}{3\times 24} = \frac{16}{24} = \frac{4\times 4}{4\times 6} = \frac{4}{6} = \frac{2\times 2}{2\times 3} = \frac{2}{3}$$

OR $\frac{48}{72} = \frac{24\times 2}{24\times 3} = \frac{2}{3}$

Now, $\frac{5}{55} = \frac{1}{11}$

Also, $\frac{21}{30} = \frac{7}{10}$

Also, $\frac{28}{49} = \frac{4}{7}$

Mixed numbers

All improper fractions may be wrtten as mixed numbers. A mixed number has a whole number and a proper fraction.

For example, $\frac{3}{2} = 1\frac{1}{2}$

Also, $\frac{7}{5} = 1\frac{2}{5}$

Also, $\frac{83}{16} = 5\frac{3}{16}$

Also, $\frac{216}{7} = 30\frac{6}{7}$

We must also be able to express a mixed number as an inproper fraction.

For example, $4\frac{3}{7} = \frac{31}{7}$

Also, $1\frac{2}{5} = \frac{7}{5}$

Example

Calculate the exact value of $\dfrac{5\frac{1}{4} - 2\frac{1}{3}}{2\frac{1}{2}}$

Solution

$$\frac{5\frac{1}{4} - 2\frac{1}{3}}{2\frac{1}{2}} = \frac{\frac{21}{4} - \frac{7}{3}}{\frac{5}{2}}$$

$$= \frac{\left(\frac{(3)(21) - (4)(7)}{12}\right)}{\frac{5}{2}} \qquad = \frac{\left(\frac{63 - 28}{12}\right)}{\frac{5}{2}}$$

$$= \frac{35/12}{5/2}$$

$$= \frac{35}{12} \div \frac{5}{2} \qquad = \frac{35}{12} \times \frac{2}{5}$$

$$= \frac{\overset{7}{\cancel{35}}}{\underset{6}{\cancel{12}}} \times \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{5}}} \qquad = \frac{7 \times 1}{1 \times 6} \qquad = \frac{7}{6}$$

$$= 1\frac{1}{6}$$

Example

Simplify $\dfrac{4\frac{1}{3}-1\frac{5}{6}}{1\frac{3}{7}\times 1\frac{2}{3}}$

Solution

$$\dfrac{4\frac{1}{3}-1\frac{5}{6}}{1\frac{3}{7}\times 1\frac{2}{3}}=\dfrac{\left(\frac{13}{3}-\frac{11}{6}\right)}{\frac{10}{7}\times\frac{5}{3}}$$

$$=\dfrac{\left(\frac{(2)(13)-(1)(11)}{6}\right)}{\frac{50}{21}} \qquad =\dfrac{\left(\frac{26-11}{6}\right)}{\frac{50}{21}}$$

$$=\dfrac{15/6}{50/21}$$

$$=\frac{15}{6}\div\frac{50}{21}$$

$$=\frac{15}{6}\times\frac{21}{50}$$

$$=\frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{6}}}\times\frac{\overset{7}{\cancel{21}}}{\underset{10}{\cancel{50}}} \qquad =\frac{3\times 7}{2\times 10}$$

$$=\frac{21}{20}$$

$$=1\frac{1}{20}$$

May 2019

Using a calculator, or otherwise, evaluate the following

$$\dfrac{2\frac{1}{4}-1\frac{3}{5}}{3}$$ (2 marks)

Solution

$$\dfrac{2\frac{1}{4}-1\frac{3}{5}}{3}=\dfrac{\frac{9}{4}-\frac{8}{5}}{3}$$

$$= \frac{\left(\frac{45-32}{20}\right)}{3}$$

$$= \frac{\left(\frac{13}{20}\right)}{3} \quad = \frac{13}{20} \div \frac{3}{1}$$

$$= \frac{13}{20} \times \frac{1}{3}$$

$$= \frac{13}{60}$$

**Please review B.O.D.M.A.S (order of algebraic operations)

<u>May 2021</u>

Using a calculator, or otherwise, calculate the EXACT value of

$$1\frac{4}{7} \div \frac{2}{3} - 1\frac{5}{7}$$ (2 marks)

<u>Solution</u>

$$1\frac{4}{7} \div \frac{2}{3} - 1\frac{5}{7} = \left(1\frac{4}{7} \div \frac{2}{3}\right) - 1\frac{5}{7}$$

$$= \left(\frac{11}{7} \div \frac{2}{3}\right) - \frac{12}{7}$$

$$= \left(\frac{11}{7} \times \frac{3}{2}\right) - \frac{12}{7}$$

$$= \frac{33}{14} - \frac{12}{7}$$

$$= \frac{33-24}{14}$$

$$= \frac{9}{14}$$

<u>May 2023</u>

Find the EXACT value of

$$\frac{5}{6} + \frac{2}{3} - \frac{12}{35} \times \frac{7}{9}$$ (2 marks)

<u>Solution</u>

$$\frac{5}{6} + \frac{2}{3} - \frac{12}{35} \times \frac{7}{9} = \left(\frac{5}{6} + \frac{2}{3}\right) - \left(\frac{12}{35} \times \frac{7}{9}\right)$$

$$= \left(\frac{5+4}{6}\right) - \left(\frac{\overset{4}{\cancel{12}}}{\underset{5}{\cancel{35}}} \times \frac{\overset{1}{\cancel{7}}}{\underset{3}{\cancel{9}}}\right)$$

$$= \frac{9}{6} - \frac{4}{15}$$

$$= \frac{45-8}{30}$$

$$= \frac{37}{30}$$

$$= 1\frac{7}{30}$$

TRY THIS (May 2022)

1. Using a calculator, or otherwise, find the EXACT value of

$$\frac{7}{8} + \frac{1}{6} \div \frac{2}{9}$$ (1 mark)

APPROXIMATIONS

An approximation is a number that is close to but not equal to an exact value.
What if we need to write an approximation for 187.264 to the nearest whole number.
Here, we look at the digit after the decimal point.
The digit after the decimal point is 2. Now, 2 is less than 5.
So, 187.|264 $\approx$ 187 to the nearest whole number.

> In general, if the digit which is to be used to decipher an approximation is 0, 1, 2, 3 or 4; such digit does NOT affect alter (change) the preceeding digit.
>
> However, if the digit which is to be used to decipher an approximation is 5 or more, such digit ALTERS (changes) the preceeding digit.

Suppose we are to approximate 34 to the nearest ten.
Here, we would have to focus on the digit in the 'ones' place in order to carry out such task. The digit in the 'ones' place is 4.
So, 3|4 $\approx$ 30 (to the nearest ten)

Suppose we are to approximate 453 to the nearest hundred.
Here, we would have to focus on the digit in the 'tens' place in order to carry out such task. The digit in the 'tens' place is 5.

So, $4|53 \approx 400 + 100 \approx 500$ (to the nearest hundred)

Suppose we are to approximate 9827 to the nearest hundred.
Here, we would have to focus on the digit in the 'tens' place in order to carry out such task. The digit in the 'tens' place is 2.
So, $98|27 \approx 9800$ (to the nearest hundred)

Approximating decimal places (d.p.)

When approximating a decimal number correct to n decimal places, we have to look at the digit in the $(n+1)$th place. If the digit in the $(n+1)$th place is greater than or equal to 5, we add 1 to the nth decimal digit. Otherwise, we do not add 1.

Suppose we are to approximate 95.617281 to 4 decimal places (d.p.)
Here, the digit in the 5th decimal place is 8, which is greater than 5.
So, $95.6172|81 \approx 95.6173$ (to 4 d.p.)

Suppose we are to approximate 126.09437 to 3 decimal places (d.p.)
Here, the digit in the 4th decimal place is 3, which is less than 5.
So, $126.094|37 \approx 126.094$ (to 3 d.p.)

Significant figures (s.f.)

In approximating a number to n significant figures, we must look at the digit value of the $(n+1)$th significant figure. If the digit of the $(n+1)$th significant figure is greater than or equal to 5, we must add 1 to the nth significant figure. Otherwise, we do not add 1.
It is important to note that ZERO (0) CANNOT be the first significant figure.
The first significant digit of a number is the first non-zero digit in the number, reading from left to right.

Consider the number 0.1275
Let's approximate 0.1275 to 3 s.f.
Now, $0.127|5 \approx 0.128$ (to 3 s.f.)

Consider the number 0.005 307 824
Let's approximate 0.005 307 824 to 5 s.f.

Now, $0.005\ 307\ 8|24 \approx 0.005\ 307\ 8$ (to 5 s.f.)

Consider the number 10.38751

Let's approximate 10.38751 to 3 s.f.

Now, $10.3|8751 \approx 10.4$ (to 3 s.f.)

Standard (scientific) form

Whenever a number is very small or very large, that number may be written in the form $a \times 10^n$, where $1 \le a < 10$ and $n \in \mathbb{Z}$.

Here, $\mathbb{Z} = \{..........,-3,-2,-1,0,1,2,3,.......\}$

Consider the number 0.0004573

Now, $0.0004573 = \dfrac{4.573}{10000}$

So, $0.0004573 = \dfrac{4.573}{1} \times \dfrac{1}{10000}$

$= 4.573 \times \dfrac{1}{10^4}$

$\therefore\ 0.0004573 = 4.573 \times 10^{-4}$

Note: $\dfrac{1}{a^m} = a^{-m}$

So, $\dfrac{1}{10^4} = 10^{-4}$

Also, $\dfrac{1}{10} = \dfrac{1}{10^1} = 10^{-1}$

Consider the number 63580000

Now, $63580000 = 6.358 \times 10000000$

So, $63580000 = 6.358 \times 10^7$

Consider the number 2.781

Now, $2.781 = 2.781 \times 1$

So, $2.781 = 2.781 \times 10^0$

$a^0 = 1$ So, $10^0 = 1$

Consider the number 0.329

Now, $0.329 = \dfrac{3.29}{10}$

$= \dfrac{3.29}{10^1}$

$$= \frac{3.29}{1} \times \frac{1}{10^1}$$

So, $0.329 = 3.29 \times 10^{-1}$

May 2023

(a) Calculate the value of $\sqrt{1-(\cos 37^\circ)^2}$ correct to 3 decimal places

(2 marks)

(b) Write 0.00527 in standard form (1 mark)

Solution

(a) $\sqrt{1-(\cos 37^\circ)^2} = \sqrt{1-0\cdot 638}$

$= \sqrt{0\cdot 362}$

$= 0\cdot 602$

(b) $0.00527 = \frac{5.27}{1000}$

$= \frac{5.27}{1} \times \frac{1}{10^3}$

$= 5\cdot 27 \times 10^{-3}$

TRY THIS (May 2022)

2. Using a calculator, or otherwise, find the

(a) EXACT value of

$$\frac{8}{0.4^3}$$ (1 mark)

(b) value of $\sqrt{26.8} - 2.5^{\frac{3}{2}}$, correct to 2 decimal places (1 mark)

May 2019

Using a calculator, or otherwise, evaluate

$2.14 \sin 75^\circ$, giving your answer to 2 decimal places (1 mark)

Solution

$2.14 \sin 75^\circ = 2.07$

FACTORIZATION

Factorization of two terms

$2x-6y=2(x-3y)$

Also, $8pq+12p=4p(2q+3)$

Also, $14m^2n-35mn=7mn(2m-5)$

Also, $9a^3b+7ab^2=ab(9a^2+7b)$

Factorization of four terms (factorization by grouping)

Consider the terms $ax+ay+bx+by$

Now, $ax+ay+bx+by=a(x+y)+b(x+y)$

$=(x+y)(a+b)$

Consider the terms $mp-4np+mq-4nq$

Now, $mp-4np+mq-4nq=p(m-4n)+q(m-4n)$

$=(m-4n)(p+q)$

Consider the terms $3xr+3xs-yr-ys$

Now, $3xr+3xs-yr-ys=3x(r+s)-y(r+s)$

$=(r+s)(3x-y)$

Factorizing quadratic expressions

Consider the quadratic expression $x^2-8x+15$.

Here, the coefficient of x^2 is +1 and the constant present is +15.

Now, $(+1)\times(+15)=+15$

Here, we need to find two integers which MULTIPLY to give +15 and (at the same time) ADD to give -8.

So, $x^2-8x+15=x^2-3x-5x+15$

$=x(x-3)-5(x-3)$

$=(x-3)(x-5)$

OR $x^2-8x+15=x^2-5x-3x+15$

$$=x(x-5)-3(x-5)$$
$$=(x-5)(x-3)$$

$$m^2-8m-105=m^2-15m+7m-105$$
$$=m(m-15)+7(m-15)$$
$$=(m-15)(m+7)$$

$$x^2-20x+100=x^2-10x-10x+100$$
$$=x(x-10)-10(x-10)$$
$$=(x-10)(x-10)$$
$$=(x-10)^2$$

Note: The expression $x^2-20x+100$ is a perfect square!

$$p^2+p-2=p^2+2p-1p-2$$
$$=p(p+2)-1(p+2)$$
$$=(p+2)(p-1)$$

Now, $2x^2-3x+1=2x^2-1x-2x+1$

$$=x(2x-1)-1(2x-1)$$
$$=(2x-1)(x-1)$$

Consider the quadratic expression $3t^2-2t-8$

Now, $3t^2-2t-8=3t^2-6t+4t-8$

$$=3t(t-2)+4(t-2)$$
$$=(t-2)(3t+4)$$

Consider the quadratic expression $4p^2+p-5$

Now, $4p^2+p-5=4p^2+5p-4p-5$

$$=p(4p+5)-1(4p+5)$$
$$=(4p+5)(p-1)$$

Consider the quadratic expression $5y^2+8y+3$

Now, $5y^2+8y+3=5y^2+5y+3y+3$

$$=5y(y+1)+3(y+1)$$
$$=(y+1)(5y+3)$$

The difference of two squares

$$\begin{aligned} a^2 - b^2 &= (a)^2 - (b)^2 \\ &= (a-b)(a+b) \end{aligned}$$

So, $$\begin{aligned} p^2 - 121 &= (p)^2 - (11)^2 \\ &= (p-11)(p+11) \end{aligned}$$

Also, $$\begin{aligned} 4t^2 - 16 &= (2t)^2 - (4)^2 \\ &= (2t-4)(2t+4) \end{aligned}$$

Also, $$\begin{aligned} 25m^2 - 1 &= (5m)^2 - (1)^2 \\ &= (5m-1)(5m+1) \end{aligned}$$

Also, $$\begin{aligned} \frac{1}{4}t^2 - 49 &= \left(\frac{1}{2}t\right)^2 - (7)^2 \\ &= \left(\frac{1}{2}t - 7\right)\left(\frac{1}{2}t + 7\right) \end{aligned}$$

Consider the quadratic expression $4x^2 + 81$

Now, $4x^2 + 81 = (2x)^2 + (9)^2$

Now, $4x^2 + 81 \neq (2x-9)(2x+9)$!

So, $4x^2 + 81$ is NOT the 'difference of two squares'!!!

May 2018

1. Factorize, completely, EACH of the following expressions

(a) $1 - 4h^2$ (1 mark)

(b) $pq - q^2 - 3p + 3q$ (2 marks)

2. Solve EACH of the following equations

(a) $\frac{3}{2}y = 12$ (1 mark)

(b) $2x^2 + 5x - 3 = 0$ (2 marks)

Solution

1. (a) $1-4h^2=(1)^2-(2h)^2$

$$=(1-2h)(1+2h)$$

(b) $pq-q^2-3p+3q=q(p-q)-3(p-q)$

$$=(p-q)(q-3)$$

2. (a) If $\frac{3}{2}y=12$, then

$$y=12\div\frac{3}{2}$$

Now, $y=\frac{12}{1}\times\frac{2}{3}\quad=\frac{24}{3}$

$\therefore\ y=8$

(b) If $2x^2+5x-3=0$, then

$$2x^2+6x-1x-3=0$$

Now, $2x(x+3)-1(x+3)=0$

So, $(x+3)(2x-1)=0$

Now, $x+3=0$ OR $2x-1=0$

$\therefore\ x=-3$ OR $x=\frac{1}{2}$

May 2022

(a) Factorize completely the quadratic expression.

$$5x^2-9x+4$$ (2 marks)

(b) Hence, solve the following equation

$$5x^2-9x+4=0$$ (1 mark)

Solution

(a) $5x^2-9x+4=5x^2-5x-4x+4$

$$=5x(x-1)-4(x-1)$$

$$=(x-1)(5x-4)$$

(b) If $5x^2-9x+4=0$, then $(x-1)(5x-4)=0$

Now, $x-1=0$ OR $5x-4=0$

$\therefore\ x=1$ OR $x=\frac{4}{5}$

TRY THIS (MAY 2017)

3. (a) Factorize the following expressions completely

(i) $6y^2 - 18xy$ (2 marks)

(ii) $4m^2 - 1$ (1 mark)

(iii) $2t^2 - 3t - 2$ (2 marks)

(b) Write as a single fraction and simplify

$$\frac{5p+2}{3} - \frac{3p-1}{4}$$ (2 marks)

TRANSPOSITIONS

A formula is an equation with two or more variables.

If $p+q=r$, then $p=r-q$

Also, if $p+q=r$, then $q=r-p$

If $a-b=c$, then $a=c+b$

If $a-b=c$, then $-b=c-a$ Now, $-(-b)=-(c-a)$

Now, $b=a-c$

If $st=u$, then $t=\frac{u}{s}$

Also, if $st=u$, then $s=\frac{u}{t}$

Consider the formula $v=u+at$.

Let's make a the subject of the formula

Now, $v-u=at$ So, $at=v-u$

So, $a=\frac{v-u}{t}$

Consider the formula $I=T+A+R$.

Let's make A the subject of the formula

Now, $T + A + R = I$

So, $A = I - T - R$

Consider the formula $2\pi f = w_1 + w_2$.

Let's make w_1 the subject of the formula

Now, $w_1 + w_2 = 2\pi f$

So, $w_1 = 2\pi f - w_2$

Consider the formula $p_2 - p_1 = \frac{2\gamma}{r}$.

Let's make p_2 the subject of the formula

Since $p_2 - p_1 = \frac{2\gamma}{r}$, then $p_2 = p_1 + \frac{2\gamma}{r}$

Now, $p_2 = \frac{p_1}{1} + \frac{2\gamma}{r}$

So, $p_2 = \frac{p_1 r + 2\gamma}{r}$

Consider the formula $T = \frac{2\pi}{w}$.

Let's make w the subject of the formula

Here $\frac{T}{1} = \frac{2\pi}{w}$

Now, $Tw = 2\pi$

So, $w = \frac{2\pi}{T}$

Consider the formula $h = \frac{2\gamma \cos\theta}{rpg}$.

Let's make γ the subject of the formula

Here $\frac{h}{1} = \frac{2\gamma \cos\theta}{rpg}$

Now, $2\gamma \cos\theta = hrpg$

So, $\gamma = \frac{hrpg}{2\cos\theta}$

Consider the formula $x = ut + \frac{1}{2}at^2$

Let's make a the subject of the formula

Now, $ut + \frac{1}{2}at^2 = x$

Now, $\frac{1}{2}at^2 = x - ut$

Multiplying both sides by 2 gives $at^2 = 2(x - ut)$

So, $at^2 = 2x - 2ut$

So, $a = \frac{2x - 2ut}{t^2}$

Consider the formula $W = \sqrt{W_1 W_2}$

Let's make W_1 the subject of the formula

Now, $(W)^2 = \left(\sqrt{W_1 W_2}\right)^2$

So, $W^2 = W_1 W_2$

So, $W_1 = \frac{W^2}{W_2}$

Consider the formula $T = \sqrt[k]{\frac{l}{g}}$

Let's make l the subject of the formula

Now, $T = \left(\frac{l}{g}\right)^{\frac{1}{k}}$

Now, $T^k = \left[\left(\frac{l}{g}\right)^{\frac{1}{k}}\right]^k$

So, $T^k = \left(\frac{l}{g}\right)^{\left(\frac{1}{k} \times \frac{k}{1}\right)}$

Now, $\frac{T^k}{1} = \frac{l}{g}$

So, $l = T^k g$

Note: (i) $\sqrt[m]{a} = a^{\frac{1}{m}}$ eg. $\sqrt[3]{8} = 8^{\frac{1}{3}}$

(ii) $(a^m)^n = a^{mn}$

Example

Transpose the formula $P - mg = \frac{mv^2}{l}$ to make v the subject.

Solution

If $P - mg = \frac{mv^2}{l}$, then

$$\frac{P - mg}{1} = \frac{mv^2}{l}$$

Now, $mv^2 = l\left(P - mg\right)$

Now, $v^2 = \frac{l\left(P - mg\right)}{m}$

$\therefore\ v = \sqrt{\frac{l\left(P - mg\right)}{m}}$

Example

Given that $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, make f the subject of the formula

Solution

If $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, then

$$\frac{u + v}{uv} = \frac{1}{f}$$

Now, $f\left(u + v\right) = uv$

$\therefore\ f = \frac{uv}{u + v}$

Example (January 2011)

Express p as the subject of the formula

$$q = \frac{p^2 - r}{t}$$ (3 marks)

Solution

If $q = \frac{p^2 - r}{t}$, then $\frac{q}{1} = \frac{p^2 - r}{t}$

Now, $p^2 - r = qt$

So, $p^2 = r + qt$

$\therefore\ p = \sqrt{r + qt}$

Example (January 2013)

Make r the subject of EACH of the following formulae:

(a) $r-h=rh$ (2 marks)

(b) $V=\pi r^2 h$ (2 marks)

Solution

(a) If $r-h=rh$, then

$$r-rh=h$$

Now, $r(1-h)=h$

$$\therefore \quad r=\frac{h}{1-h}$$

(b) If $V=\pi r^2 h$, then

$$r^2=\frac{V}{\pi h}$$

$$\therefore \quad r=\sqrt{\frac{V}{\pi h}}$$

Example (May 2013)

(a) Make C the subject of the formula $F=\frac{9}{5}C+32$ (2 marks)

(b) Given that $F=113$, calculate the value of C (1 mark)

Solution

(a) If $F=\frac{9}{5}C+32$, then

$$F=\frac{9C}{5}+32$$

Now, $\frac{9C}{5}=F-32$

So, $\frac{5}{9}\left(\frac{9C}{5}\right)=\frac{5}{9}(F-32)$

$$\therefore \quad C=\frac{5}{9}(F-32)$$

(b) When $F=113$, $C=\frac{5}{9}(113-32)$

Now, $C=\frac{5}{9}\times\frac{81}{1}$

So, $C = \frac{5}{\not{9}_{1}} \times \frac{\not{81}^{9}}{1} = \frac{5 \times 9}{1 \times 1}$

$\therefore \quad C = 45$

Example (May 2022)

Make v the subject of the formula

$$w = \frac{5 + v}{v - 3}$$ (3 marks)

Solution

If $w = \frac{5 + v}{v - 3}$, then $\frac{w}{1} = \frac{5 + v}{v - 3}$

Now, $w(v - 3) = 5 + v$

Now, $wv - 3w = 5 + v$

Now, $wv - v = 5 + 3w$

Now, $v(w - 1) = 5 + 3w$

$\therefore \quad v = \frac{5 + 3w}{w - 1}$

Example (May 2021)

Two quantities, n and T, are related as follows:

$$n = \sqrt{T}$$

(a) Find the value of n when $T = 49$ (1 mark)

(b) Make T the subject of the formula (1 mark)

Solution

(a) When $T = 49$, $n = \sqrt{49}$

$\therefore \quad n = 7$

(b) If $n = \sqrt{T}$, then $n^2 = \left(\sqrt{T}\right)^2$

$\therefore \quad T = n^2$

TRY THIS (May 2018)

4. The quantities F, m, u, v and t are related according to the formula

$$F = \frac{m(v-u)}{t}$$

(a) Find the value of F when $m=3, u=-1, v=2$ and $t=1$ (1 mark)

(b) Make v the subject of the formula (2 marks)

SOLVING EQUATIONS

Consider the equation $4+x=11$

Now, $4+x-4=11-4$

So, $x=7$

Consider the equation $12p=96$

Now, $\frac{12p}{12}=\frac{96}{12}$

So, $p=8$

Consider the equation $5=3y+2$

Now, $3y+2=5$

So, $3y+2-2=5-2$

So, $3y=3$

Now, $\frac{3y}{3}=\frac{3}{3}$

So, $y=1$

Consider the equation $7x-15=3x+1$

Now, $7x-15-3x=3x+1-3x$

So, $4x-15=1$

Now, $4x-15+15=1+15$

So, $4x=16$

Now, $\frac{4x}{4}=\frac{16}{4}$

So, $x=4$

If $3x-4(1-3x)=2x-(x+1)$, then

$3x-4+12x=2x-x-1$

Now, $15x-4=x-1$

So, $15x-4+4=x-1+4$

Now, $15x=x+3$

Now, $15x-x=x+3-x$

So, $14x=3$

Now, $\frac{14x}{14}=\frac{3}{14}$

Now, $x=\frac{3}{14}$

If $\frac{3x-1}{2}-\frac{x-2}{3}=6$, then multiplying throughout by 6 gives

$$\left(\frac{6}{1}\times\frac{3x-1}{2}\right)-\left(\frac{6}{1}\times\frac{x-2}{3}\right)=6(6)$$

Now, $3(3x-1)-2(x-2)=36$

Now, $9x-3-2x+4=36$

So, $7x+1=36$

So, $7x+1-1=36-1$

Now, $7x=35$

So, $\frac{7x}{7}=\frac{35}{7}$

$\therefore\ x=5$

If $\frac{5x}{7}-\frac{x}{14}=\frac{9}{7}$, then

$$\left(\frac{14}{1}\times\frac{5x}{7}\right)-\left(\frac{14}{1}\times\frac{x}{14}\right)=\left(\frac{14}{1}\times\frac{9}{7}\right)$$

Now, $\frac{70x}{7}-\frac{14x}{14}=\frac{126}{7}$

So, $10x-x=18$

Now, $9x=18$

So, $x=\frac{18}{9}$

$\therefore\ x=2$

The GENERAL FORM of a quadratic equation is $ax^2+bx+c=0$, where a, b and c are integers AND $a\neq 0$.

Examples of quadratic equations are $3x^2=12$, $x^2+2x=0$ and $x^2-5x-6=0$

Now, if $ax^2+bx+c=0$, then $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

This is known as the QUADRATIC FORMULA.

Example

Solve the equation $3x^2=10x+8$

Solution

If $3x^2=10x+8$, then

$3x^2-10x-8=0$

Now, $3x^2-12x+2x-8=0$

So, $3x(x-4)+2(x-4)=0$

Now, $(x-4)(3x+2)=0$

So, $x-4=0$ OR $3x+2=0$

Now, $x=4$ OR $3x=-2$

$\therefore$ $x=4$ OR $x=-\frac{2}{3}$

OR If $3x^2=10x+8$, then $3x^2-10x-8=0$

In the general form $ax^2+bx+c=0$, $a=3$, $b=-10$ and $c=-8$

Now, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Now, $x=\frac{-(-10)\pm\sqrt{(-10)^2-4(3)(-8)}}{2(3)}$

Now, $x=\frac{10\pm\sqrt{100+96}}{6}$

So, $x=\frac{10\pm\sqrt{196}}{6}$

Then $x=\frac{10\pm14}{6}$

Now, $x=\frac{10+14}{6}$ OR $x=\frac{10-14}{6}$

So, $x=\frac{24}{6}$ OR $x=\frac{-4}{6}$

$\therefore$ $x=4$ OR $x=-\frac{2}{3}$

Note: It is easier to solve equations of the general form $ax^2+bx+c=0$ if the terms in the left hand side can be factorized. We may use the quadratic formula to solve these equations if the terms in the left hand side CANNOT be expressed in the form $(x+p)(x+q)$, where p and q are integers.

Consider the quadratic equation $x^2+4x+1=0$.

In the general form $ax^2+bx+c=0$, $a=1$, $b=4$ and $c=1$.

Now, $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Now, $x=\dfrac{-4\pm\sqrt{(4)^2-4(1)(1)}}{2(1)}$

So, $x=\dfrac{-4\pm\sqrt{16-4}}{2}$

Then $x=\dfrac{-4\pm\sqrt{12}}{2}$

Now, $x=\dfrac{-4\pm3.46}{2}$

Now, $x=\dfrac{-4+3.46}{2}$ OR $x=\dfrac{-4-3.46}{2}$

So, $x=\dfrac{-0.54}{2}$ OR $x=\dfrac{-7.46}{2}$

$\therefore$ $x=-0.27$ OR $x=-3.73$

Example (May 2019)

Solve the equation

$$\frac{3}{7x-1}+\frac{1}{x}=0$$ (3 marks)

Solution

If $\dfrac{3}{7x-1}+\dfrac{1}{x}=0$, then

$\dfrac{3x+7x-1}{x(7x-1)}=0$

Now, $\dfrac{10x-1}{x(7x-1)}=0$

Now, $10x-1=0$

So, $10x=1$

$\therefore$ $x=\dfrac{1}{10}$

SOLVING SIMULTANEOUS EQUATIONS

Case 1: Two linear equations

Consider the pair of linear equations

$$x - y = -10$$
$$x + y = -2$$

Method of elimination

$$x - y = -10 \text{............} eq(1)$$
$$x + y = -2 \text{..............} eq(2)$$

Adding eq(1) and eq(2) gives $2x = -12$

Now, $x = \frac{-12}{2}$

So, $x = -6$

Substituting $x = -6$ into eq(1) gives $-6 - y = -10$

Now, $-y = -10 + 6$

So, $-y = -4$

Now, $y = \frac{-4}{-1}$

So, $y = 4$

$\therefore$ $x = -6$ and $y = 4$

OR
$$x - y = -10 \text{............} eq(1)$$
$$x + y = -2 \text{..............} eq(2)$$

Eq(1) minus eq(2) gives $-y - y = -10 - (-2)$

Now, $-2y = -10 + 2$

So, $-2y = -8$

Now, $y = \frac{-8}{-2}$

So, $y = 4$

Substituting $y = 4$ into eq(2) gives $x + 4 = -2$

Now, $x = -2 - 4$

So, $x = -6$

$\therefore$ $x = -6$ and $y = 4$

Method of substitution

$x - y = -10$............eq(1)

$x + y = -2$..............eq(2)

From eq(1), $x = -10 + y$

So, $x = y - 10$

Substituting $x = y - 10$ into eq(2) gives $y - 10 + y = -2$

Now, $2y - 10 = -2$

So, $2y = -2 + 10$

So, $2y = 8$

Then $y = \frac{8}{2}$

So, $y = 4$

Substituting $y = 4$ into eq(1) gives $x - 4 = -10$

Now, $x = -10 + 4$

So, $x = -6$

$\therefore$ $x = -6$ and $y = 4$

Example

Solve the pair of simultaneous equations

$$2x - 7y = -13$$
$$3x + 5y = -4$$

Solution (elimination)

$2x - 7y = -13$.......eq(1)

$3x + 5y = -4$..........eq(2)

Multiplying eq(1) throughout by 5 gives $10x - 35y = -65$...........eq(3)

Multiplying eq(2) throughout by 7 gives $21x + 35y = -28$...........eq(4)

Adding eq(3) and eq(4) gives $10x + 21x = -65 + (-28)$

Now, $31x = -93$

So, $x = \frac{-93}{31}$

So, $x = -3$

Substituting $x = -3$ into eq(1) gives $2(-3) - 7y = -13$

Now, $-6 - 7y = -13$

So, $-7y = -13 + 6$

So, $-7y=-7$

So, $y=\frac{-7}{-7}$

So, $y=1$

$\therefore \quad x=-3$ and $y=1$

OR $\quad 2x-7y=-13 ....eq(1)$

$3x+5y=-4 ....eq(2)$

Multiplying eq(1) throughout by 3 gives $6x-21y=-39 ....eq(3)$

Multiplying eq(2) throughout by 2 gives $6x+10y=-8 ....eq(4)$

Eq(3) minus eq(4) gives $-21y-(+10y)=-39-(-8)$

Now, $-21y-10y=-39+8$

So, $-31y=-31$

Now, $y=\frac{-31}{-31}$

So, $y=1$

Substituting $y=1$ into eq(2) gives $3x+5(1)=-4$

Now, $3x+5=-4$

So, $3x=-4-5$

So, $3x=-9$

So, $x=\frac{-9}{3}$

So, $x=-3$

$\therefore \quad x=-3$ and $y=1$

Solution (substitution)

$2x-7y=-13 ....eq(1)$

$3x+5y=-4 ....eq(2)$

From eq (1), $2x=7y-13$

So, $x=\frac{7y-13}{2}$

Substituting $x=\frac{7y-13}{2}$ into eq(2) gives $3\left(\frac{7y-13}{2}\right)+5y=-4$

So, $\frac{3(7y-13)}{2}+5y=-4$

So, $\frac{21y-39}{2}+5y=-4$

Multiplying throughout by 2 gives $2\left(\frac{21y-39}{2}\right)+2(5y)=2(-4)$

Now, $21y-39+10y=-8$

So, $31y=-8+39$

So, $31y=31$

So, $y=\frac{31}{31}$

So, $y=1$

Substituting $y=1$ into eq(1) gives $2x-7(1)=-13$

Now, $2x-7=-13$

So, $2x=-13+7$

So, $2x=-6$

So, $x=\frac{-6}{2}$

So, $x=-3$

$\therefore$ $x=-3$ and $y=1$

Example (May 2016)

Solve the simultaneous equations:

$2x+y=3$

$5x-2y=12$

Solution

$2x+y=3$.............eq(1) $\times 2$

$5x-2y=12$.........eq(2) $\times 1$

$4x+2y=6$.......eq(1)

$5x-2y=12$......eq(2)

Adding eq(1) and eq(2) gives $9x=18$

Now, $x=2$

Substituting $x=2$ into eq(1) gives $2(2)+y=3$

Now, $4+y=3$

Now, $y=3-4$

So, $y=-1$

$\therefore \quad x=2$ and $y=-1$

TRY THIS (May 2014)

5. Solve the following simultaneous equations:

$$2x + 3y = 9$$
$$3x - y = 8$$

(3 marks)

Case 2: One linear equation, one quadratic equation

Consider the pair of equations

$$y = 2x^2 + 1$$
$$y = 7x + 5$$

Now, $y = 2x^2 + 1$........eq(1)

$y = 7x + 5$.........eq(2)

Substituting $y = 7x + 5$ into eq(1) gives $7x + 5 = 2x^2 + 1$

Now, $2x^2 + 1 = 7x + 5$

So, $2x^2 + 1 - 7x - 5 = 0$

Now, $2x^2 - 7x - 4 = 0$

Now, $2x^2 - 8x + 1x - 4 = 0$

So, $2x(x-4) + 1(x-4) = 0$

Now, $(x-4)(2x+1) = 0$

Now, $x - 4 = 0$ OR $2x + 1 = 0$

So, $x = 4$ OR $2x = -1$

So, $x = 4$ OR $x = -\frac{1}{2}$

Using eq(2), when $x = 4$, $y = 7(4) + 5$

So, $y = 28 + 5$

So, $y = 33$

When $x = -\frac{1}{2}$, $y = 7\left(-\frac{1}{2}\right) + 5$

So, $y = -\frac{7}{2} + 5 \; = -\frac{7}{2} + \frac{10}{2}$

So, $y=\frac{3}{2}$

$\therefore$ when $x=4,\ y=33$ and when $x=-\frac{1}{2},\ y=\frac{3}{2}$

Example

Solve the pair of simultaneous equations

$$x-y=2$$
$$xy=15$$

Solution

$$x-y=2.........\text{eq}(1)$$
$$xy=15.......\text{eq}(2)$$

From eq(1), $x=y+2$

Substituting $x=y+2$ into eq(2) gives $(y+2)y=15$

Now, $y^2+2y=15$

Now, $y^2+2y-15=0$

Now, $y^2+5y-3y-15=0$

So, $y(y+5)-3(y+5)=0$

Now, $(y+5)(y-3)=0$

So, $y+5=0$ OR $y-3=0$

So, $y=-5$ OR $y=3$

Using the equation $x=y+2$, when $y=-5$, $x=-5+2$

So, $x=-3$

When $y=3$, $x=3+2$

So, $x=5$

$\therefore$ when $x=-3,\ y=-5$ and when $x=5,\ y=3$

Example (May 2011)

Solve the pair of simultaneous equations

$$y=x^2-x+3$$
$$y=6-3x$$

(5 marks)

Solution

$$y=x^2-x+3...........\text{eq}(1)$$
$$y=6-3x................\text{eq}(2)$$

Substituting $y=6-3x$ into eq(1) gives $6-3x=x^2-x+3$

Now, $x^2-x+3=6-3x$

Now, $x^2-x+3-6+3x=0$

So, $x^2+2x-3=0$

Now, $x^2+3x-1x-3=0$

So, $x(x+3)-1(x+3)=0$

Now, $(x+3)(x-1)=0$

So, $x+3=0$ OR $x-1=0$

$\therefore$ $x=-3$ OR $x=1$

Using eq(2), when $x=-3$, $y=6-3(-3)$ $=6+9$ So, $y=15$

When $x=1$, $y=6-3(1)$ $=6-3$ So, $y=3$

$\therefore$ when $x=-3$, $y=15$ and when $x=1$, $y=3$

<u>Example (May 2012)</u>

(a) Solve the pair of simultaneous equations

$y=8-x$

$2x^2+xy=-16$ (5 marks)

(b) State, giving a reason for your answer, whether the line $y=8-x$ is a tangent to the curve $2x^2+xy=-16$ (2 marks)

<u>Solution</u>

$y=8-x$.................eq(1)

$2x^2+xy=-16$........eq(2)

Substituting $y=8-x$ into eq(2) gives $2x^2+x(8-x)=-16$

Now, $2x^2+8x-x^2=-16$

So, $2x^2+8x-x^2+16=0$

Now, $x^2+8x+16=0$

Now, $x^2+4x+4x+16=0$

So, $x(x+4)+4(x+4)=0$

Now, $(x+4)(x+4)=0$

So, $x+4=0$ OR $x+4=0$

So, $x=-4$ OR $x=-4$

Using eq(1), when $x = -4$, $y = 8 - (-4)$ $= 8 + 4$ So, $y = 12$

$\therefore$ when $x = -4$, $y = 12$

(b) There is ONE point of contact between the straight line $y = 8 - x$ and the curve $2x^2 + xy = -16$. So, the line $y = 8 - x$ is a tangent to the curve $2x^2 + xy = -16$.

TRY THIS (June 2009)

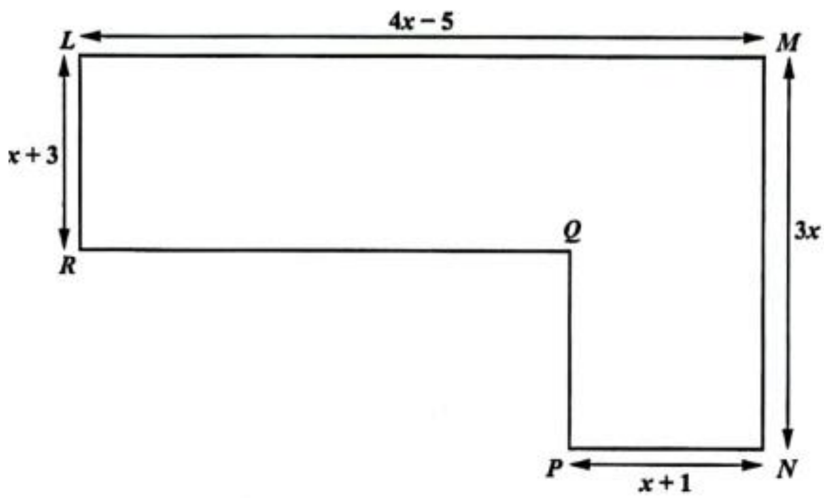
6. Solve the pair of simultaneous equations

$y = 4 - 2x$

$y = 2x^2 - 3x + 1$ (4 marks)

Example (May 2023)

The diagram below shows a compound shape, *LMNPQR*, made from two rectangles. The lengths in the diagram, which are written in terms of x, are in centimetres.



(a) Find an expression, in terms of x, for the length

i) PQ (1 mark)

ii) RQ (1 mark)

(b) Given that the TOTAL area of the shape is 414 cm^2, show that

$x^2 + x - 72 = 0$ (4 marks)

Solution

(a) i) $PQ = 3x - (x + 3)$

$= 3x - x - 3$

$= 2x - 3$

(a) ii) $RQ = 4x - 5 - (x+1)$

$= 4x - 5 - x - 1$

$= 3x - 6$

(b) Since the TOTAL area of the shape $= 414 \text{ cm}^2$, then

$(LR \times RQ) + (MN \times PN) = 414$

Now, $\left[(x+3)(3x-6)\right] + \left[3x(x+1)\right] = 414$

So, $3x^2 - 6x + 9x - 18 + 3x^2 + 3x = 414$

Now, $6x^2 + 6x - 18 = 414$

Now, $6x^2 + 6x - 18 - 414 = 0$

So, $6x^2 + 6x - 432 = 0$

Dividing throughout by 6 gives $x^2 + x - 72 = 0$

TRY THIS (May 2021)

7. Ally is x years. Jim is 5 years older than Ally and Chris is twice as old as Ally.

(a) Write expressions in terms of x for Jim's age and Chris' age

Jim's age ....

Chris' age ....

(2 marks)

(b) In two years' time, the product of Ally's age and Chris' age will be the same as the square of Jim's PRESENT age.

Show that the equation $x^2 - 4x - 21 = 0$ represents the information given above.

(3 marks)

(c) Calculate Ally's present age (2 marks)

WAGE BILL, CONSUMER ARITHMETIC

Example (May 2023)

Haresh works at a call centre for 35 hours each week. He is paid an hourly rate of $11.20.

(a) Calculate the amount of money Haresh earns in a FOUR-WEEK month.

(2 marks)

(b) In a certain week, Haresh works 8 hours overtime. Overtime hours are paid at $1\frac{1}{2}$ times the usual rate of \$11.20 per hour.

Find the TOTAL amount of money Haresh is paid for THAT WEEK. (2 marks)

Solution

(a) Amount earned in a FOUR-WEEK period $= 4 \times 35 \times \$11.20$

$= \$1,568$

(b) Amount earned for 35 hours $= 35 \times \$11.20$

$= \$392$

Amount earned for 8 hours OVERTIME $= 8 \times \$11.20 \times 1.5$

$= \$134.40$

TOTAL amount earned for THAT WEEK $= \$392 + \$134.40$

$= \$526.40$

TRY THIS (May 2019)

8. Irma's take-home pay is \$4 320 per fortnight (every two weeks). Each fortnight Irma's pay is allocated according to the following table.

Item	**Amount Allocated**
Rent	$\$x$
Food	\$629
Other living expenses	$\$2x$
Savings	\$1 750
Total	**\$4 320**

(a) What is Irma's ANNUAL take-home pay? (Assume she works 52 weeks in any given year) (1 mark)

(b) Determine the amount of money that Irma allocates for rent each month (3 marks)

(c) All of Irma's savings is used to pay her son's university tuition cost, which is \$150 000.

If Irma's pay remain the same and she saves the same amount each month, what is the MINIMUM number of years that she must work in order to save enough money to cover her son's tuition cost? (2 marks)

Example (May 2022)

Children go to a Rodeo camp during the Easter holiday. Ms Rekha buys bananas and oranges for the children at the camp.

(a) Bananas cost \$3.85 per kilogram. Ms Rekha buys 25 kg of bananas and receives a discount of 12%. How much money does she spend on bananas? (2 marks)

(b) Ms Rekha spends \$165.31, inclusive of a sales tax of 15%, on oranges. Calculate the original price of the oranges. (2 marks)

(c) The ratio of the number of bananas to the number of oranges is 2:3. Furthermore, there are 24 oranges than bananas. Calculate the number of bananas that Ms Rekha bought. (2 marks)

Solution

(a) $25 \times \$3.85 = \$96.25$

Now, $\frac{12}{100} \times \$96.25 = \$11.55$

Now, $\$96.25 - \$11.55 = \$84.70$

$\therefore$ Ms Rekha spent \$84.70 on bananas

(b) Ms Rekha spends \$165.31, inclusive of a sales tax of 15%, on oranges.

Let x represent the original price of the oranges.

Now, $100\% + 15\% = 115\%$

Now, $\frac{115}{100} \times x = \$165.31$

Now, $x = \frac{\$165.31}{1} \div \frac{115}{100}$

So, $x = \frac{\$165.31}{1} \times \frac{100}{115}$

So, $x = \$143.75$

∴ the original price of the oranges is $143.75

(c) $3-2=1$ part

So, 1 part is equivalent to 24

Now, $2 \times 24 = 48$

∴ Ms Rekha bought 48 bananas

Example (May 2017)

John's monthly electricity bill is based on the number of kWh of electricity that he consumes for that month. He is charge \$5.10 per kWh of electricity consumed. For the month of March 2016, two meter readings are displayed in the table below.

	Meter Readings (kWh)
Beginning 01 March	0 3 0 1 1
Ending 31 March	0 3 3 0 7

(a) Calculate the TOTAL amount that John pays for electricity consumption for the month of March 2016 (2 marks)

(b) For the next month, April 2016, John pays \$2351.10 for electricity consumption. Determine his meter reading at the end of April 2016 (2 marks)

Solution

(a)

```
 0 3 3 0 7      Meter reading ending March 31
 0 3 0 1 1 –    Meter reading beginning March 01
 ---------
 0 0 2 9 6
 ---------
```

John used 296 kWh of electricity for the month of March 2016.

Now, $296 \times \$5.10 = \$1509.60$

∴ John pays \$1509.60 for electricity consumption for the month of March 2016

b) $\frac{\$2351.10}{\$5.10} = 461$

John used 461 kW of electricity for the month of April 2016.

Now,

0 3 3 0 7		Meter reading beginning April 01
0 0 4 6 1	+	Number of kW used for April 2016
0 3 7 6 8		

$\therefore$ John's meter reading at the end of April 2016 is 0 3 7 6 8

TRY THIS (May 2017)

9. A store is promoting a new mobile phone under two plans: Plan A and Plan B. The plans are advertised as shown in the table below.

	Plan A	**Plan B**
Deposit	\$400	\$600
Monthly instalment	\$65	\$80
Number of months to repay	12	6
Tax on ALL payments	0%	5%

(a) Calculate the TOTAL cost of a phone under Plan A (2 marks)

(b) Determine which of the two plans, A or B, is the better deal. Justify your answer. (2 marks)

VARIATION

Direct variation

If y is directly proportional to x, then we write $y \propto x$

Here, y varies directly with x.

So, $y = kx$

Here, k is called the constant of proportionality

Example (May 2022)

The height, h, of an object is directly proportional to the square root of its perimeter, p.

(a) Write an equation showing the relationship between h and p (1 mark)

(b) Given that $h = 5.4$ when $p = 1.44$, determine the value of h when $p = 2.89$

(2 marks)

Solution

(a) Here, $h = k\sqrt{p}$

(b) When $h = 5.4$, $p = 1.44$

So, $5.4 = k\sqrt{1.44}$

Now, $1.2k = 5.4$

Now, $k = \dfrac{5.4}{1.2}$

$\therefore \quad k = 4.5$

Now, $h = 4.5\sqrt{p}$

When $p = 2.89$, $h = 4.5 \times \sqrt{2.89}$

So, $h = 4.5 \times 1.7$

$\therefore \quad h = 7.65$

Inverse variation

If y is inversely proportional to x, then we write $y \propto \dfrac{1}{x}$

We can also say that y is directly proportional to $\dfrac{1}{x}$

Here, y varies inversely with x.

Now, $y = k\left(\dfrac{1}{x}\right)$

So, $y = \dfrac{k}{x}$

Here, k is called the constant of proportionality

Example (May 2019)

The quantity P varies inversely is the square of V.

(a) Using the letters P, V and k, form an EQUATION connecting the quantities P and V (1 mark)

(b) Given that $V = 3$ and $P = 4$, determine the positive value of V when $P = 1$ (2 marks)

Solution

(a) Here, $P \propto \frac{1}{V^2}$

Now, $P = k\left(\frac{1}{V^2}\right)$

$\therefore \quad P = \frac{k}{V^2}$

(b) When $V = 3$, $P = 4$

So, $4 = \frac{k}{3^2}$

Now, $\frac{4}{1} = \frac{k}{9}$

Now, $k = (4)(9)$

So, $k = 36$

Now, $P = \frac{36}{V^2}$

When $P = 1$, $1 = \frac{36}{V^2}$

Now, $\frac{1}{1} = \frac{36}{V^2}$

Now, $V^2 = 36$

Now, $V = \sqrt{36}$

$\therefore \quad V = 6$ when $P = 1$

TRY THIS (May 2016)

10. The table below shows corresponding values of the variables x and y, where y varies directly as x.

x	6	10	t
y	3	u	9

Calculate the values of t and u (3 marks)

TRY THIS (May 2013)

11. The incomplete table below shows one pair of values for A and R where A is directly proportional to the square of R.

A	36		196
R	3	5	

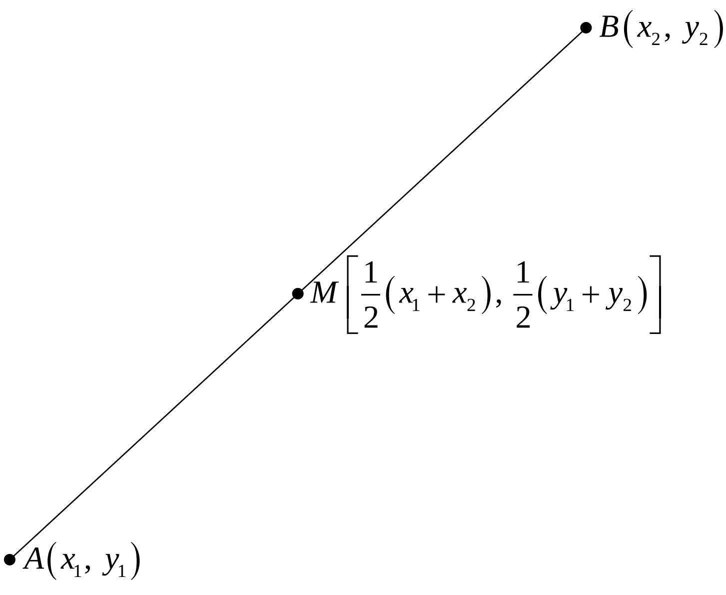
(a) Express A in terms of R and a constant k (1 mark)

(b) Calculate the value of the constant k (2 marks)

(c) Copy and complete the table (2 marks)

COORDINATE GEOMETRY

Midpoint of a line segment



If $A(x_1, y_1)$ and $B(x_2, y_2)$ are the end points of the line AB, the MIDPOINT of AB, denoted by M; is given by

$$M = \left[\frac{1}{2}(x_1 + x_2), \frac{1}{2}(y_1 + y_2)\right]$$

Example

Find the midpoint of the straight line joining the points $C(-3, 1)$ and $D(5, 9)$

Solution

The midpoint of $CD = \left[\frac{1}{2}(-3+5), \frac{1}{2}(1+9)\right]$

$$= \left[\frac{1}{2}(2), \frac{1}{2}(10)\right]$$

$$= (1, 5)$$

Example

Find the midpoint of the straight line joining the points $A(-7, -3)$ and $B(-1, 2)$

Solution

The midpoint of $AB = \left[\frac{1}{2}(-7+-1), \frac{1}{2}(-3+2)\right]$

$$= \left[\frac{1}{2}(-8), \frac{1}{2}(-1)\right]$$

$$= \left(-4, -\frac{1}{2}\right)$$

The length of a line segment

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are the endpoints of a straight line, the LENGTH of AB, denoted $|AB|$, is given by

$$|AB| = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

Example

Find the length of the straight line joining the points $M(-4, 4)$ and $N(5, 7)$.

Solution

$$|MN| = \sqrt{(7-4)^2 + (5--4)^2}$$

$$= \sqrt{3^2 + 9^2} \quad = \sqrt{9+81}$$

$$= \sqrt{90}$$

$= 9.49$ units

Example

Find the length of the straight line joining the points $J(-6, 8)$ and $K(-2, 5)$.

Solution

$$|JK| = \sqrt{(5-8)^2 + (-2--6)^2}$$

$$= \sqrt{(-3)^2 + (-2+6)^2} = \sqrt{(-3)^2 + 4^2}$$

$$= \sqrt{9+16}$$

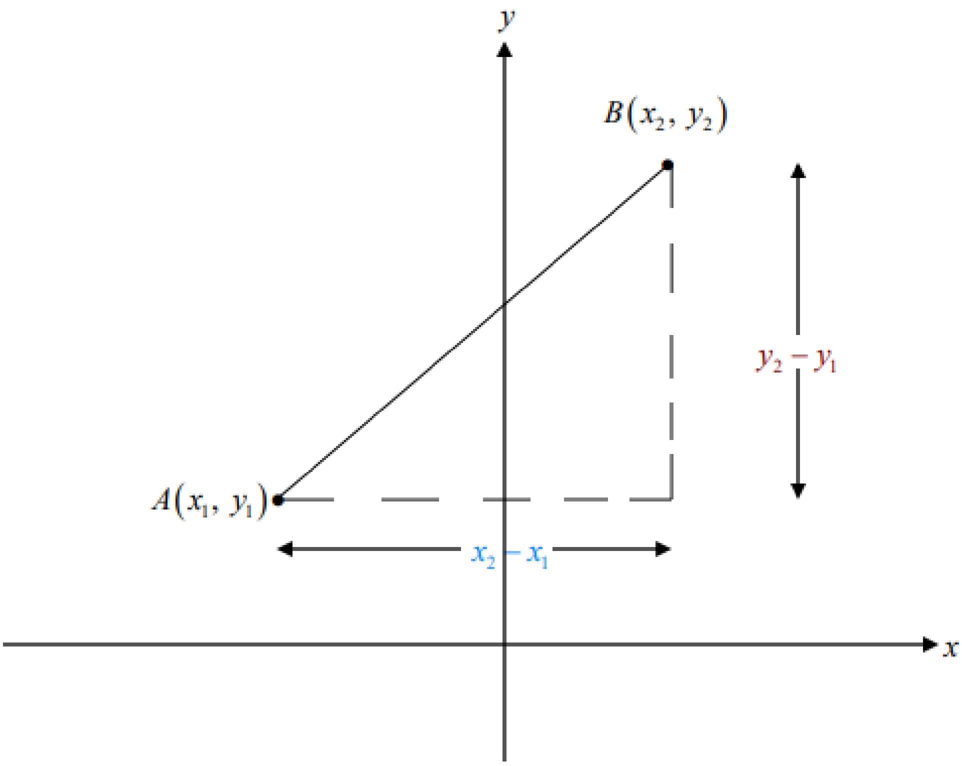
$$= \sqrt{25}$$

$= 5$ units

Finding the gradient of a straight line

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are the endpoints of a straight line, the GRADIENT of AB, denoted m, is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



The gradient of the straight line AB is the change in the value of the y-coordinates divided by the change in the value of the x-coordinates.

Example

Find the gradient of the line joining the points $P(-3, -2)$ and $Q(5, 6)$.

Solution

$$\text{Gradient of } PQ = \frac{6-(-2)}{5-(-3)}$$
$$= \frac{8}{8}$$
$$= 1$$

Example

Find the gradient of the line joining the points $A(3, -1)$ and $B(9, 4)$.

Solution

$$\text{Gradient of } AB = \frac{4-(-1)}{9-3}$$
$$= \frac{5}{6}$$

Note: (i) The gradient of a straight line which is parallel to the $x-$axis is ZERO

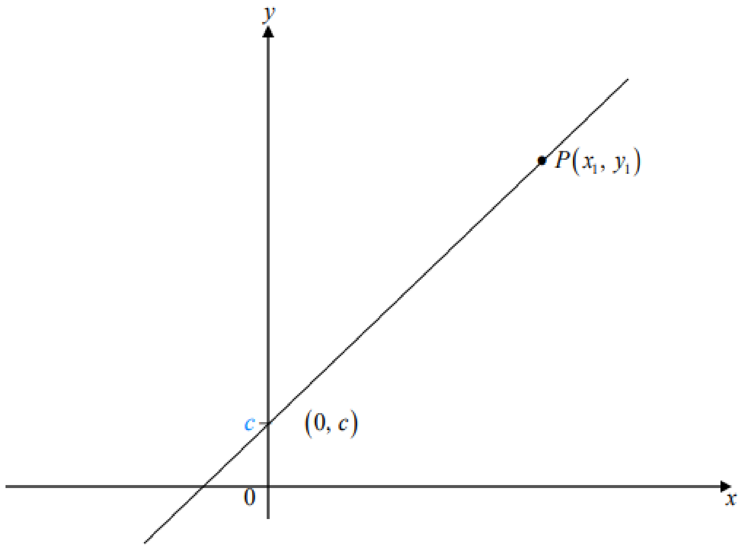
(ii) The gradient of a straight line which is parallel to the $y-$axis is INFINITE

Finding the equation of a straight line

The equation of a straight line may be found if either of the following is known:

(i) the coordinates of a point on the line and its gradient OR

(ii) the coordinates of two points on the line



Example

Find the equation of the straight line which passes through the point $(-3, 1)$ with gradient -1.

Solution

The equation of the straight line is given by $y = mx + c$

Since the line passes through the point $(-3, 1)$ and the gradient of the line is -1,

then $1 = -1(-3) + c$ Here, we need to find the value of c

$x-$coordinate of point on line

gradient of the line

$y-$coordinate of point on line

Now, $1 = 3 + c$

So, $1 - 3 = c$

So, $c = -2$

Now, the equation of the straight line is $y = -x - 2$

Example

Find the equation of the straight line which passes through the point $(-9, 3)$ with gradient $\frac{2}{5}$.

Solution

The equation of the straight line is given by $y = mx + c$

Since the line passes through the point $(-9, 3)$ and the gradient of the line is $\frac{2}{5}$,

then $3 = \frac{2}{5}(-9) + c$

Now, $3 = -\frac{18}{5} + c$

So, $3 + \frac{18}{5} = c$

So, $c = \frac{15}{5} + \frac{18}{5}$

Therefore $c = \frac{33}{5}$

$\therefore$ the equation of the straight line is $y = \frac{2}{5}x + \frac{33}{5}$

Example

Find the equation of the straight line which passes through the points $P(6, -1)$ and $Q(4, 5)$

Solution

$$\text{Gradient of the straight line} = \frac{5-(-1)}{4-6} = \frac{6}{-2} = -3$$

Now, the equation of the straight line is given by $y = mx + c$.

So, $-1 = (-3)(6) + c$

Now, $-1 = -18 + c$

Now, $-1 + 18 = c$

So, $c = 17$

$\therefore$ the equation of the straight line is $y = -3x + 17$

Note: If two (or more) points lie on a straight line, we may use the coordinates of ANY of the points to find the equation of the straight line.

From the example above, the coordinates of the point $Q(4, 5)$ may be used to find the equation of the straight line.

Using the point $Q(4, 5)$, the equation of the straight line PQ is given by

$5 = (-3)(4) + c$

So, $5 = -12 + c$

Now, $5 + 12 = c$

So, $c = 17$

$\therefore$ the equation of the straight line PQ is $y = -3x + 17$

Example

Find the equation of the straight line which passes through the points $(-7, -5)$ and $(2, 3)$

Solution

$$\text{Gradient of the straight line} = \frac{3-(-5)}{2-(-7)} = \frac{8}{9}$$

Now, the equation of the straight line is given by $y = mx + c$.

So, $-5 = \left(\frac{8}{9}\right)(-7) + c$

Now, $-5 = -\frac{56}{9} + c$

Now, $-5 + \frac{56}{9} = c$

So, $-\frac{45}{9} + \frac{56}{9} = c$

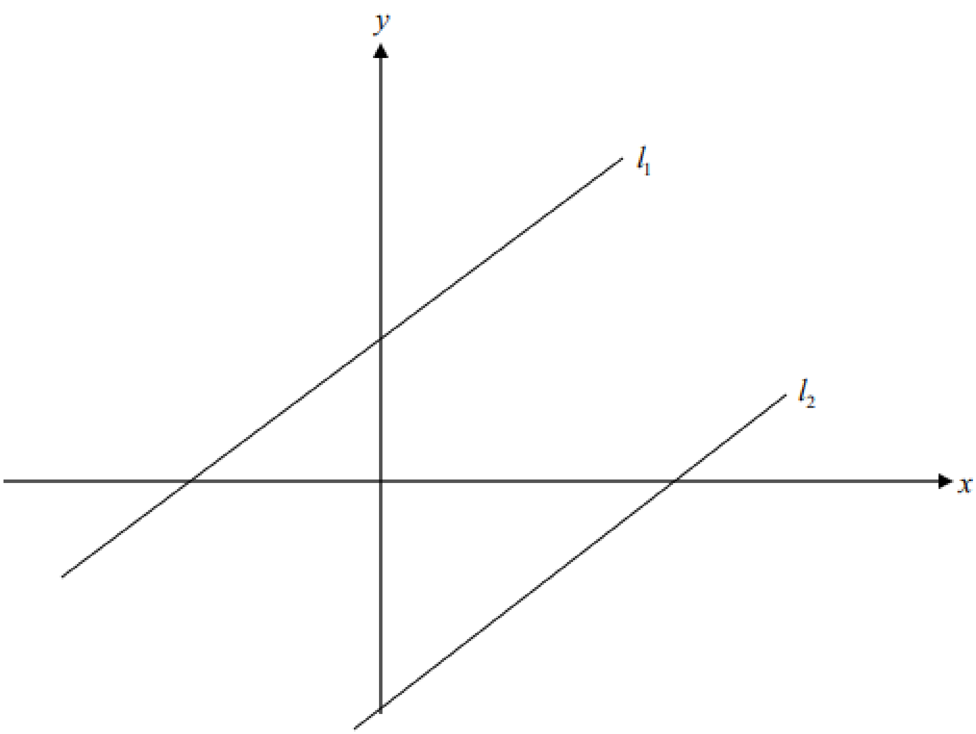
So, $c = \frac{11}{9}$

$\therefore$ the equation of the straight line is $y = \frac{8}{9}x + \frac{11}{9}$

Parallel lines

Two lines are PARALLEL if they have the SAME GRADIENT.

So, if l_1 and l_2 are two parallel lines and their respective gradients are m_1 and m_2, then $m_1 = m_2$.



Parallel lines do NOT intersect.

Perpendicular lines

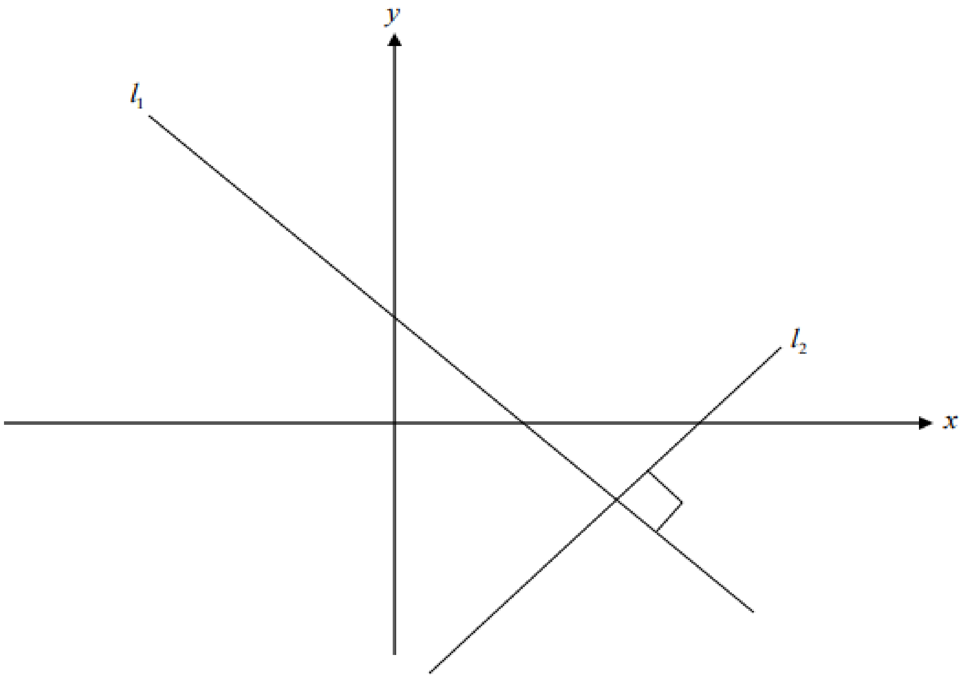
Two lines are PERPENDICULAR to each other if the product of their gradients is -1.

So, if l_1 and l_2 are two perpendicular lines and their respective gradients are m_1 and m_2, then $$m_1 \times m_2 = -1$$

Now, $m_1 = -\frac{1}{m_2}$

Now, $m_2 = -\frac{1}{m_1}$

Here, the gradient of either line is the NEGATIVE RECIPROCAL of the gradient of the other line.



Example

Find the equation of the straight line which passes through the point $(-5, 7)$ and perpendicular to the straight line $x - 3y + 1 = 0$.

Solution

If $x - 3y + 1 = 0$, then $-3y = -x - 1$

Now, $y = \frac{1}{3}x + \frac{1}{3}$ $\{$in the form $y = mx + c\}$

So, the gradient of the straight line $x - 3y + 1 = 0$ is $\frac{1}{3}$.

The gradient of a line perpendicular to the line $x-3y+1=0$ is -3.

Now, the equation of the line which passes through the point $(-5, 7)$ and perpendicular to the straight line $x-y+1=0$ is in the form $y=mx+c$.

Now, $7=(-3)(-5)+c$

So, $7=15+c$

Now, $7-15=c$

So, $c=-8$

$\therefore$ the equation of the straight line is $y=-3x-8$

(i)	At the point where a straight line cuts the $X-\text{AXIS}$, $y=0$
(ii)	At the point where a straight line cuts the $Y-\text{AXIS}$, $x=0$

Example (May 2019)

The equation of a straight line is given as

$$\frac{x}{3}+\frac{y}{7}=1$$

This line crosses the $y-$axis at Q

(a) Determine the coordinates of Q (1 mark)

(b) What is the gradient of this line? (2 marks)

Solution

(a) $x=0$ at the point Q

Substituting $x=0$ into the equation $\frac{x}{3}+\frac{y}{7}=1$ gives $\frac{0}{3}+\frac{y}{7}=1$

Now, $\frac{y}{7}=1$

Now, $7\left(\frac{y}{7}\right)=7(1)$

So, $y=7$

$\therefore$ the coordinates of Q are $(0, 7)$

(b) Since the equation of the straight line is $\frac{x}{3}+\frac{y}{7}=1$, then by multiplying the equation throughout by 21, we get

$$21\left(\frac{x}{3}\right)+21\left(\frac{y}{7}\right)=21(1)$$

Now, $7x+3y=21$

Now, $3y=-7x+21$

So, $y=-\frac{7}{3}x+7$ $\{$in the form $y=mx+c\}$

$\therefore$ the gradient of the line is $-\frac{7}{3}$

Past paper (January 2011)

The equation of a straight line is given by

$$3y=2x-6$$

Determine

(a) the gradient of the line (2 marks)

(b) the equation of the line which is perpendicular to $3y=2x-6$, and passes through the point $(4, 7))$ (3 marks)

Solution

(a) If $3y=2x-6$, then

$y=\frac{2}{3}x-2$ $\{$in the form $y=mx+c\}$

$\therefore$ the gradient of the line is $\frac{2}{3}$

(b) The gradient of a line perpendicular to the line $3y=2x-6$ is $-\frac{3}{2}$.

Now, the equation of the line which passes through the point $(4, 7)$ and perpendicular to the straight line $3y=2x-6$ is in the form $y=mx+c$.

Now, $7=\left(-\frac{3}{2}\right)(4)+c$

So, $7=-6+c$

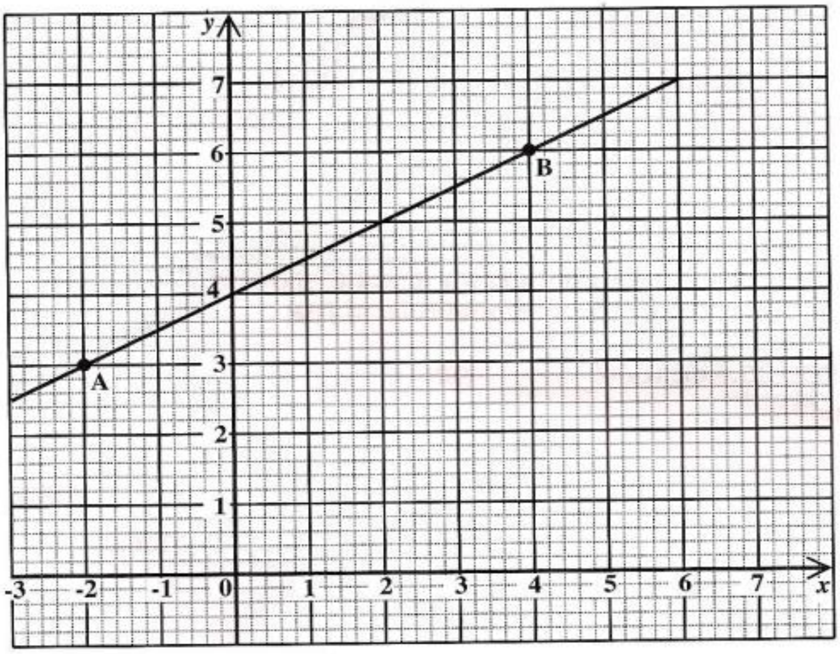
Now, $7+6=c$

So, $c=13$

$\therefore$ the equation of the straight line is $y=-\frac{3}{2}x+13$

Example (May 2011)

The diagram below shows the line segment which passes through the points A and B.



Determine

(i) the coordinates of A and B **(2 marks)**

(ii) the gradient of the line segment AB **(2 marks)**

(iii) the equation of the line which passes through A and B. **(2 marks)**

Solution

(i) The coordinates of A are $(-2, 3)$ and the coordinates of B are $(4, 6)$

(ii) Gradient of $AB = \dfrac{6-3}{4-(-2)} = \dfrac{3}{6}$

$= \dfrac{1}{2}$

(iii) Using the point $A(-2, 3)$, the equation of AB is given in the form $y = mx + c$

Now, $3 = \frac{1}{2}(-2) + c$

So, $3 = -1 + c$

Now, $3 + 1 = c$

So, $c = 4$

$\therefore$ the equation of the line which passes through A and B is $y = \frac{1}{2}x + 4$

Example (May 2013)

$A(-1, 4)$ and $B(3, 2)$ are the end points of a line segment AB. Determine

(a) the gradient of AB (2 marks)

(b) the coordinates of the midpoint of AB (2 marks)

(c) the equation of the perpendicular bisector of AB (3 marks)

Solution

(a) Gradient of $AB = \frac{2-4}{3-(-1)} = \frac{-2}{4}$

$$= -\frac{1}{2}$$

(b) Midpoint of $AB = \left[\frac{1}{2}(-1+3), \frac{1}{2}(4+2)\right]$

$$= \left[\frac{1}{2}(2), \frac{1}{2}(6)\right]$$

$$= (1, 3)$$

(c) Since the gradient of AB is $-\frac{1}{2}$, the gradient of the perpendicular bisector of AB is 2.

Since the perpendicular bisector of AB passes through the point $(1, 3)$, the equation of the perpendicular bisector of AB is in the form $y = mx + c$, where $m = 2$.

Now, $3 = 2(1) + c$

So, $3 = 2 + c$

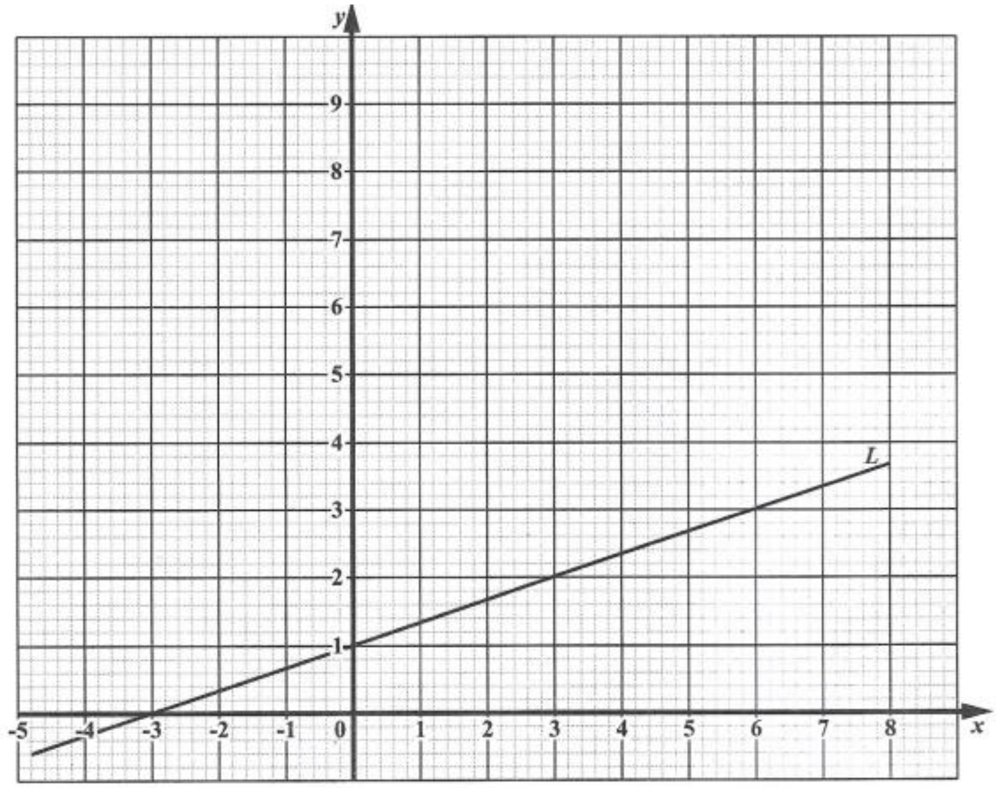
Now, $3 - 2 = c$

So, $c = 1$

$\therefore$ the perpendicular bisector of AB is $y = 2x + 1$

Example (May 2022)

The line L is shown on the grid below.



(a) Write the equation of the line L in the form $y = mx + c$ (2 marks)

(b) The equation of a different line, Q, is $y = -2x + 8$.

i) Write down the coordinates of the point where Q crosses the $x-$axis (1 mark)

ii) Write down the coordinates of the point where Q crosses the $y-$axis (1 mark)

iii) On the grid above, draw the graph of the line Q (1 mark)

(c) Complete the statement below.

According to the graph, the solution of the system of equations consisting of L and Q is .... (1 mark)

Solution

(a) Using the points $(0, 1)$ and $(3, 2)$, the gradient of $L = \frac{2-1}{3-0}$

$= \frac{1}{3}$

Using the coordinates of the point $(0, 1)$, the equation of L is given by

$$1 = \left(\frac{1}{3}\right)(0) + c$$

Now, $1 = 0 + c$

So, $c = 1$

$\therefore$ the equation of L is $y = \frac{1}{3}x + 1$

(b) i) Substituting $y = 0$ into the equation $y = -2x + 8$ gives $0 = -2x + 8$

Now, $-2x = -8$

So, $x = 4$

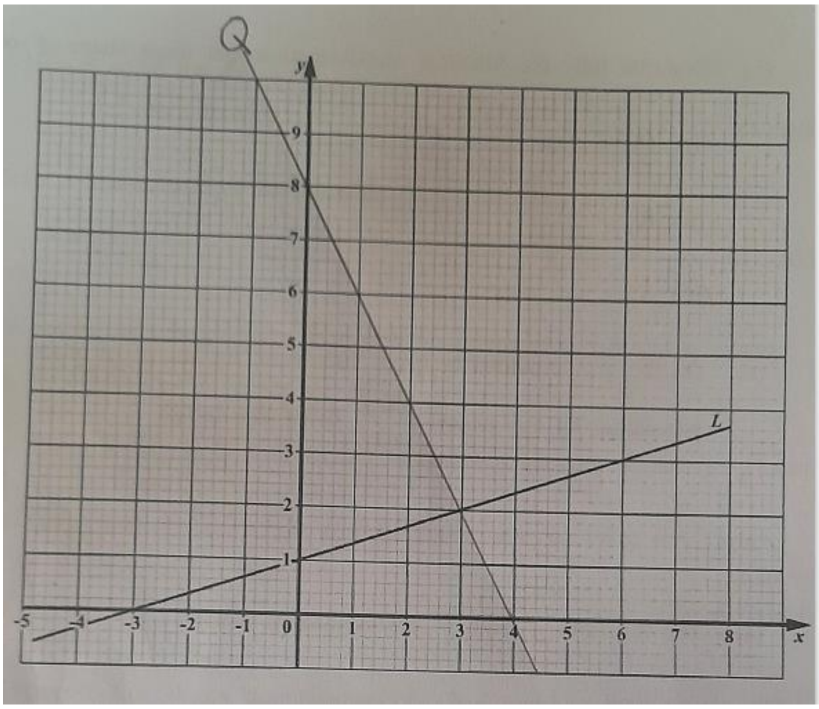
$\therefore$ the coodinates of the point where Q crosses the $x-$axis are $(4, 0)$

ii) Substituting $x = 0$ into the equation $y = -2x + 8$ gives $y = -2(0) + 8$

So, $y = 8$

$\therefore$ the coodinates of the point where Q crosses the $y-$axis are $(0, 8)$

(b) iii)

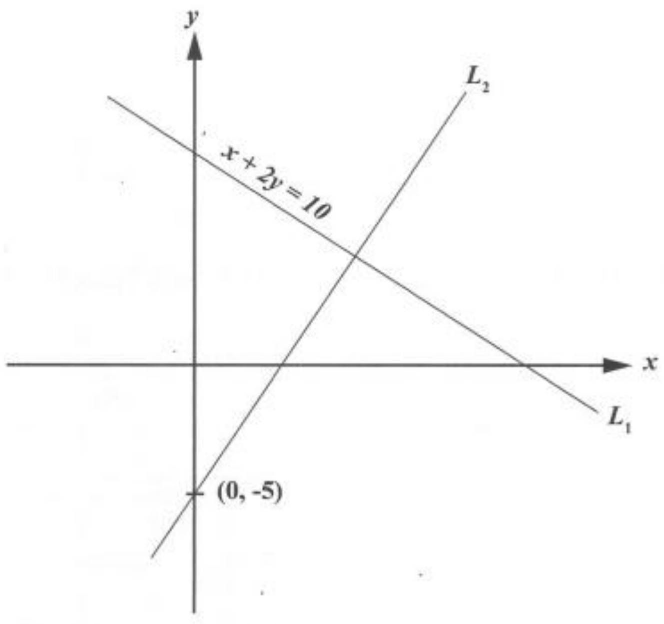


(c) Complete the statement below.

According to the graph, the solution of the system of equations consisting of L and Q is $(3, 2)$

Example (May 2021)

The diagram below shows two lines, L_1 and L_2. The equation of the line L_1 is $x+2y=10$. The line L_2 passes through the point $(0, -5)$ and is PERPENDICULAR to L_1.



(a) Express the equation of the line L_1 in the form $y = mx + c$ (1 mark)

(b) State the gradient of the line L_1 (1 mark)

(c) Hence, determine the equation of the line L_2 (2 marks)

Solution

(a) If $x+2y=10$, then $2y=-x+10$

Now, $y=-\frac{1}{2}x+5$ (in the form $y=mx+c$)

(b) The gradient of the line L_1 is $-\frac{1}{2}$

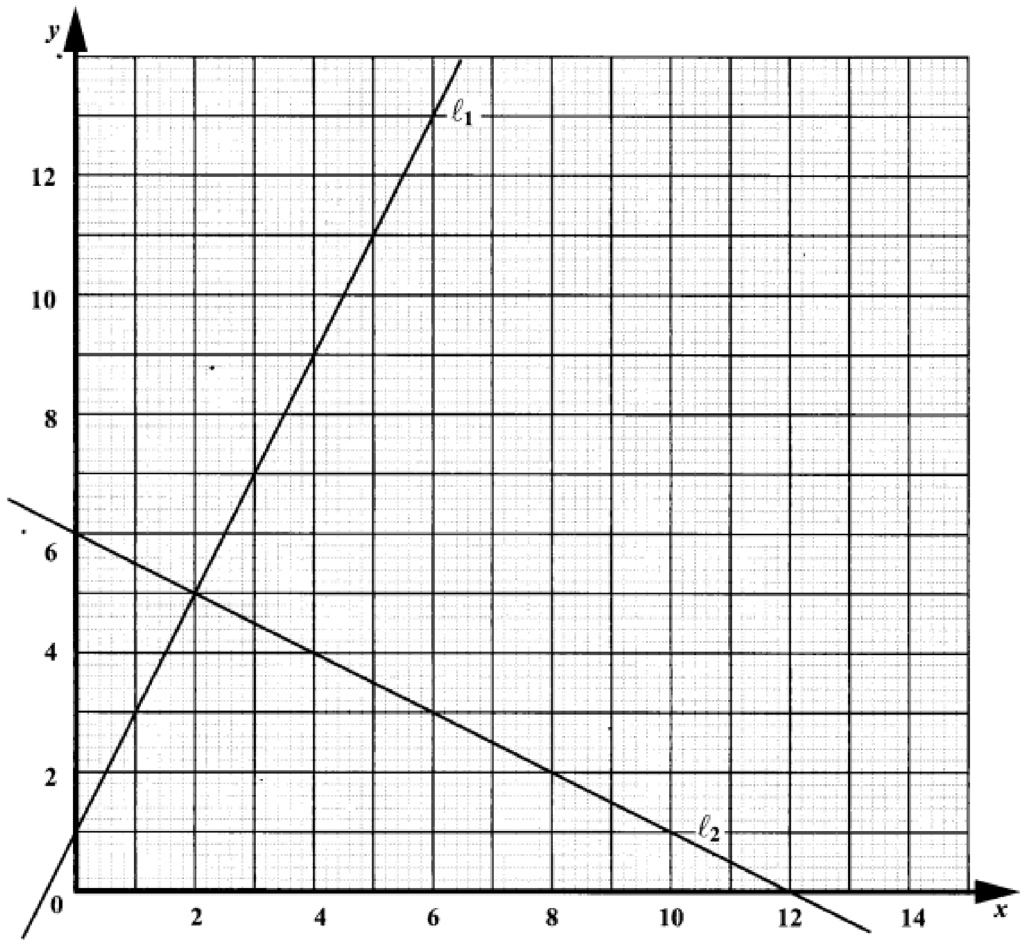
(c) Since L_1 and L_2 are perpendicular to each other and the gradient of L_1 is $-\frac{1}{2}$, then the gradient of the line L_2 is 2.

Also, the $y-$intercept of L_2 is -5

So, the equation of the line L_2 is $y = 2x - 5$

TRY THIS (May 2017)

12. The graph below shows two straight lines, l_1 and l_2. Line l_1 intercept the $y-$axis at $(0, 1)$. Line l_2 intercepts the x and y axes at $(12, 0)$ and $(0, 6)$ respectively.



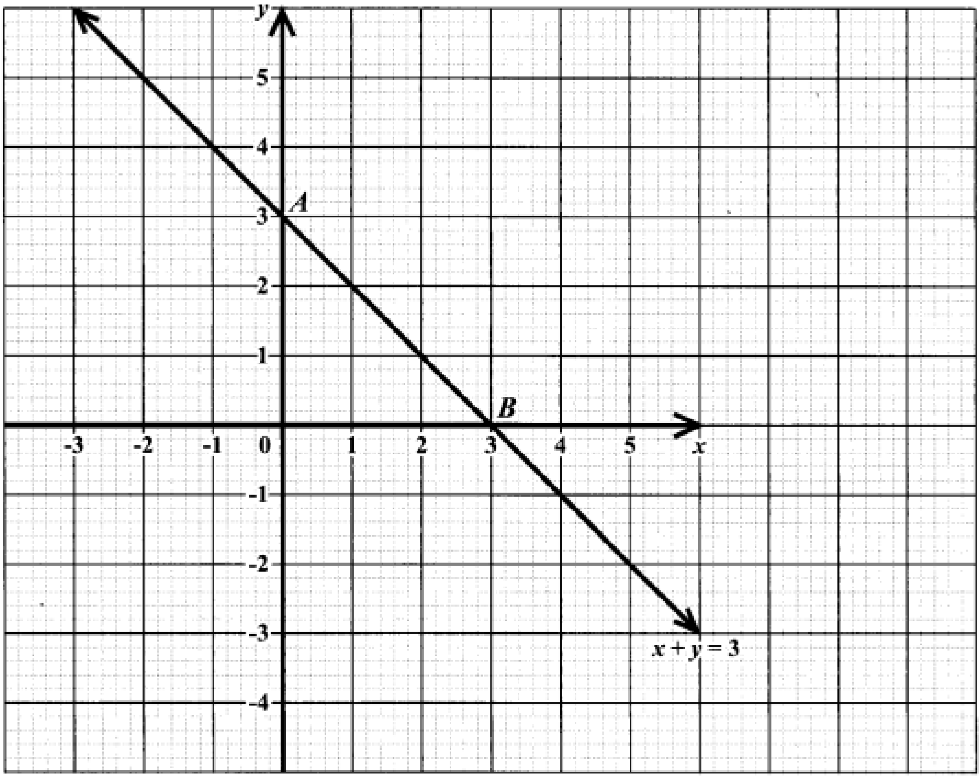
(a) Calculate the gradient of the lines l_1 and l_2 (2 marks)

(b) Determine the equation of the line l_1 (2 marks)

(c) What is the relationship between l_1 and l_2? Give a reason for your answer (2 marks)

TRY THIS (May 2016)

13. The diagram below shows the graph of the straight line $x + y = 3$



Determine the equation of the line which is

(a) parallel to the line $x + y = 3$ and passes through the origin (2 marks)

(b) perpendicular to the line $x + y = 3$ and passes through the midpoint of AB (2 marks)

STATISTICS

Example (May 2016)

Twenty bags of sugar were weighed. The weight, to the nearest kg, are as follows:

3	38	17	33	28
12	43	38	31	30
11	8	23	18	26
50	22	35	39	5

(a) Complete the frequency table for the data shown above.

Weight (kg)	Tally	Number of Bags
1–10		
11–20		
21–30		
31–40		
41–50		

(4 marks)

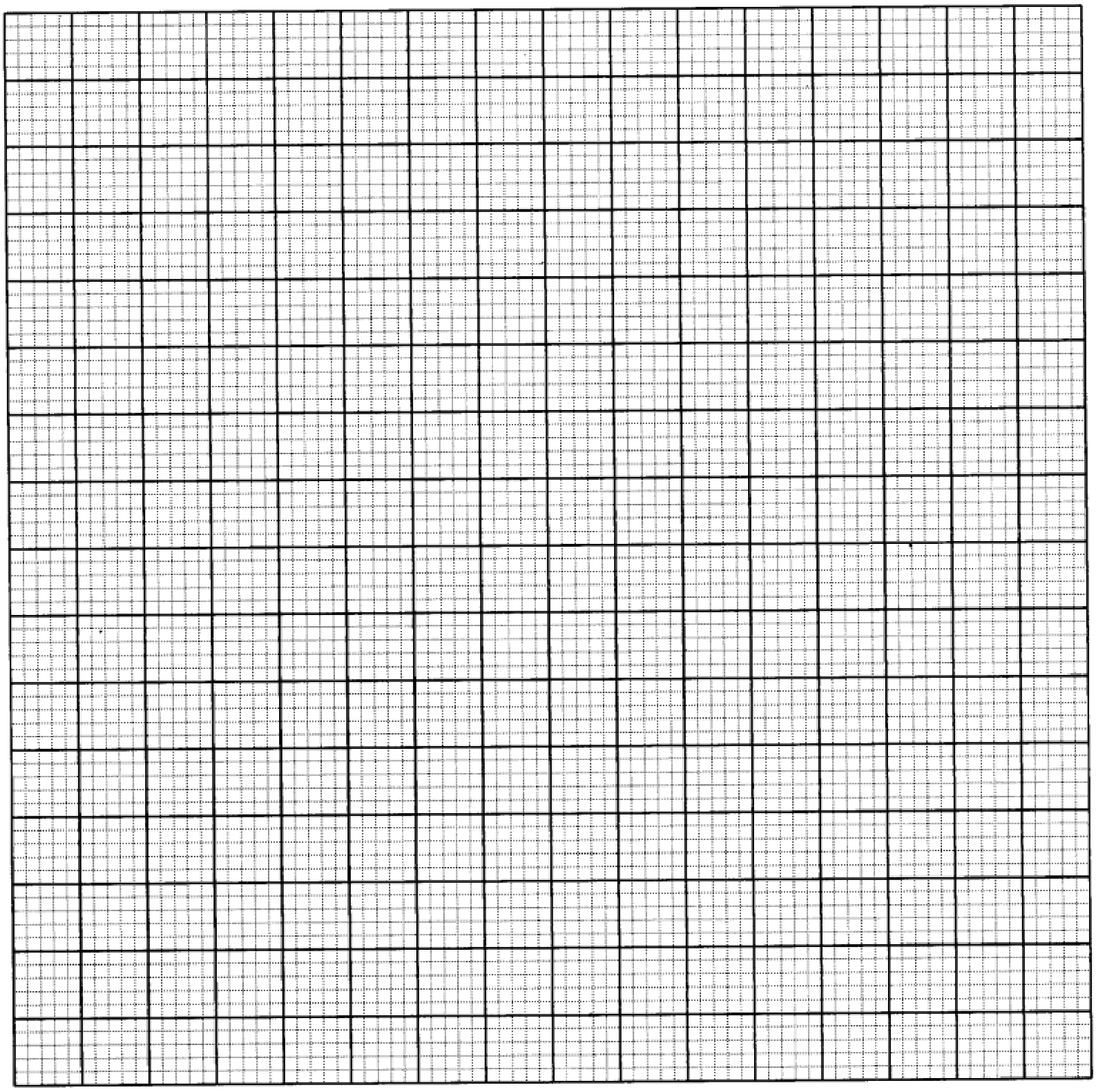
(b) For the class interval $21-30$, state:

(i) the upper class boundary (1 mark)

(ii) the class width (1 mark)

(iii) the class midpoint (1 mark)

(c) On the grid below, using a scale of 2 cm to represent 10 kg on the $x-$axis, and 1 cm to represent 1 bag on the $y-$axis, draw a histogram to represent the data contained in your frequency table above. (4 marks)



<u>Solution</u>

(a)

Weight (kg)	Tally	Number of Bags
1–10	III	3
11–20	IIII	4
21–30	~~IIII~~	5
31–40	~~IIII~~ I	6
41–50	II	2

(b) For the class interval $21-30$, state:

(i) the upper class boundary is 30.5

(ii) the lower class boundary is 20.5

Now, the class width $=$ upper class boundary $-$ lower class boundary

$$= 30.5 - 20.5$$

$$= 10$$

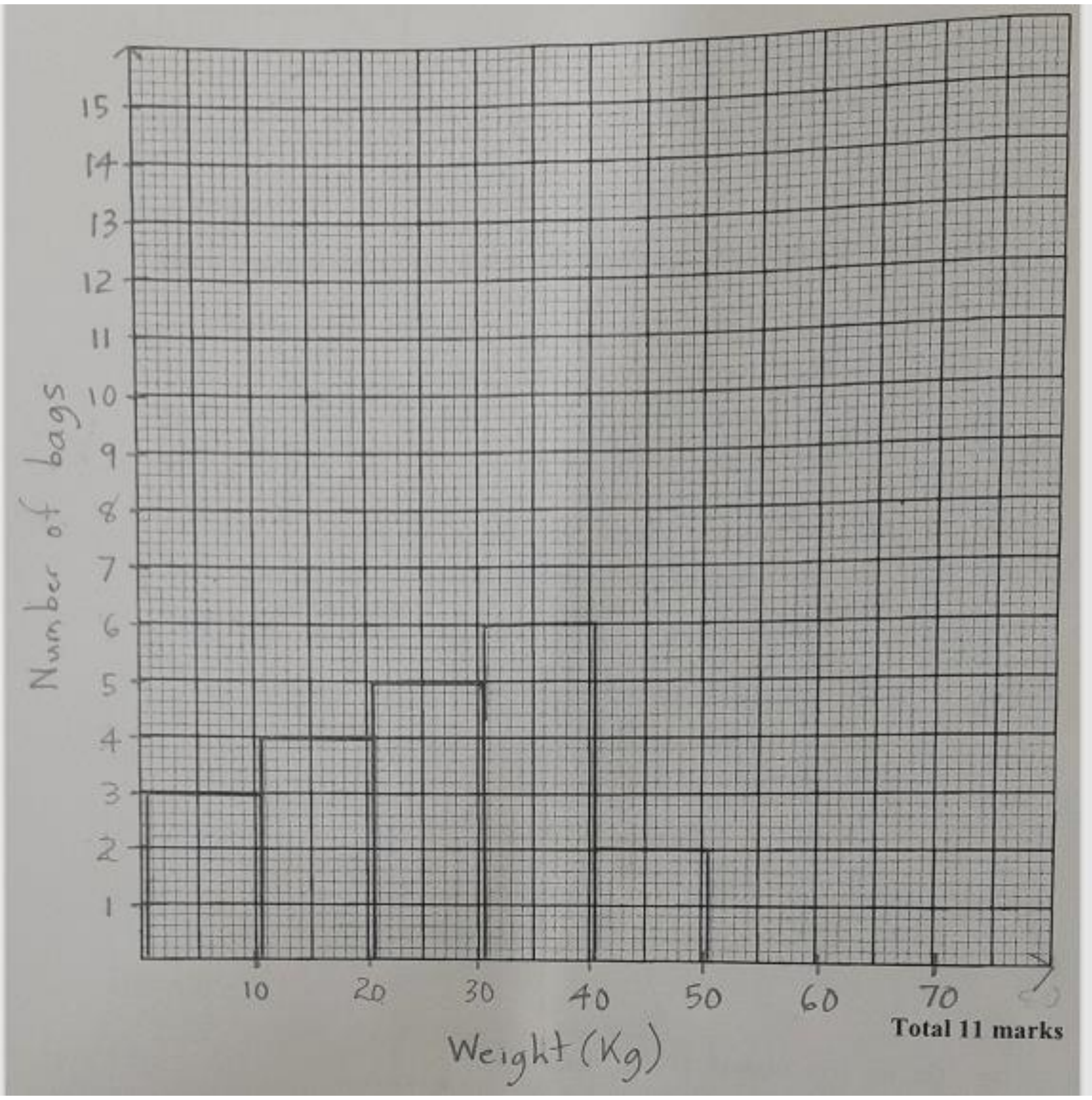
(iii) the class midpoint $= \dfrac{\text{lower class boundary} + \text{upper class boundary}}{2}$

$$= \frac{20.5 + 30.5}{2}$$

$$= \frac{51}{2}$$

$$= 25.5$$

(c)



TRY THIS (May 2014)

14 A class of 30 students counted the number of books in their bags on a certain day. The number of books in EACH bag is shown below.

5	4	6	3	2	1	7	4	5	3
6	5	4	3	7	6	2	5	4	5
5	7	5	4	3	2	1	6	3	4

(a) Copy and complete the frequency table for the data shown above.

Number of Books (x)	Tally	Frequency (f)	$f \times x$
1	||	2	2
2	|||	3	6
3			
4			
5			
6			
7			

(4 marks)

(b) State the modal number of books in the bags of the sample of students.

(c) Using the table in (a) above, or otherwise, calculate

(i) the TOTAL number of books (2 marks)

(ii) the mean number of books per bag (2 marks)

(d) Determine the probability that a student chosen at random has LESS THAN 4 books in his/her bag. (2 marks)

Example (May 2019)

The cumulative frequency distribution of the volume of petrol needed to fill the tanks of 150 different vehicles is shown below.

Volume (litres)	Cumulative Frequency
11–20	24
21–30	59
31–40	101
41–50	129
51–60	150

(a) For the class 21 – 30, determine the

(i) lower class boundary (1 mark)

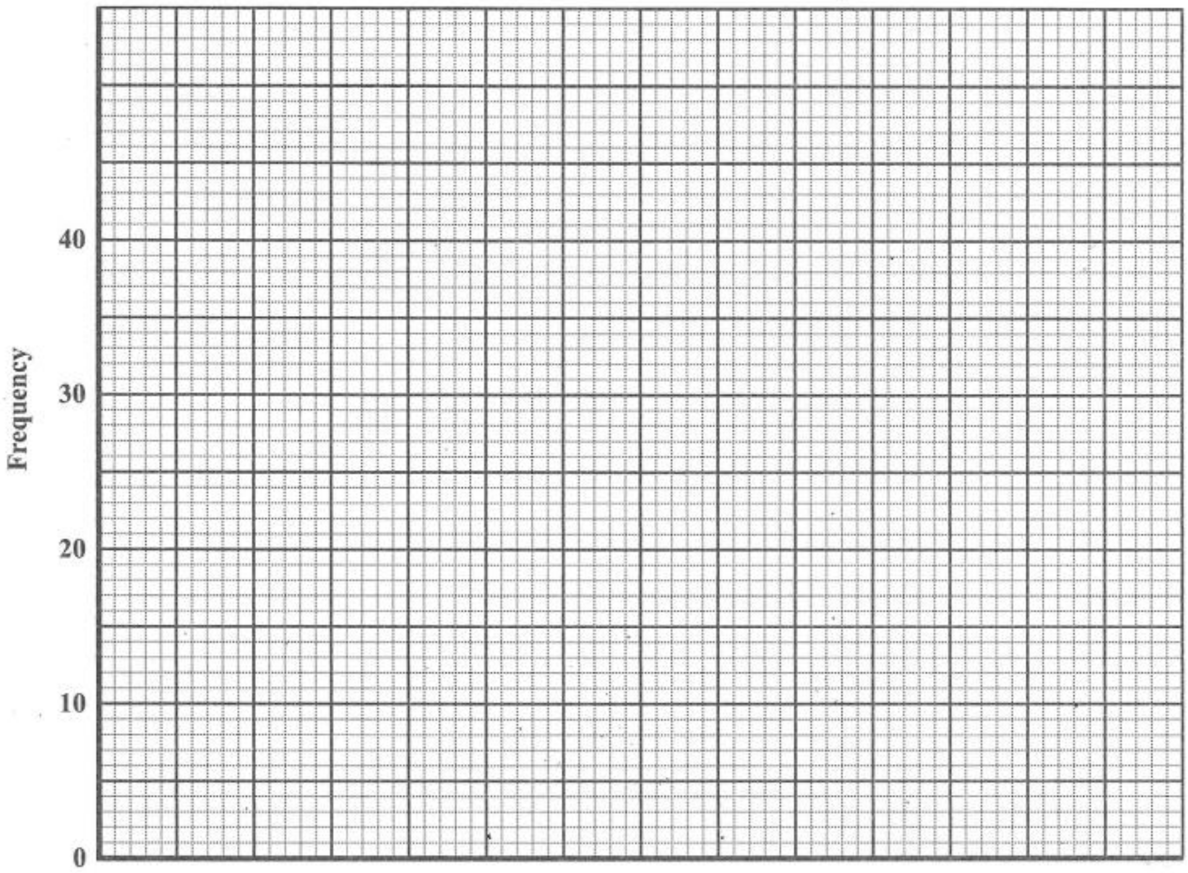
(ii) class width (1 mark)

(b) How many vehicles were recorded in the class 31 – 40? (1 mark)

(c) A vehicle is chosen at random from the 150 vehicles. What is the probability that the volume of petrol needed to fill its tank is MORE than 50.5 litres? Leave your answer as a fraction. (2 marks)

(d) Byron estimates the median amount of petrol to be 43.5 litres. Explain why Byron's estimate is INCORRECT. (1 mark)

(e) On the partially labelled grid below, construct a histogram to represent the distribution of the volume of petrol needed to fill the tanks of the 150 vehicles.



(3 marks)

Solution

(a) (i) The lower class boundary for the class $21-30$ is 20.5

(ii) The upper class boundary for the class $21-30$ is 30.5

Now, the class width is $30.5-20.5=10$

(b) Number of vehicles recorded in the class $31-40 = 101-59$

$= 42$

(c) Probability that more than 50.5 litres of petrol needed to fill tank is

$$\frac{150-129}{150} = \frac{21}{150}$$

$$= \frac{7}{50}$$

(d) The total frequency, $n = 150$

Now, the median is the $\frac{1}{2}n^{th}$ term.

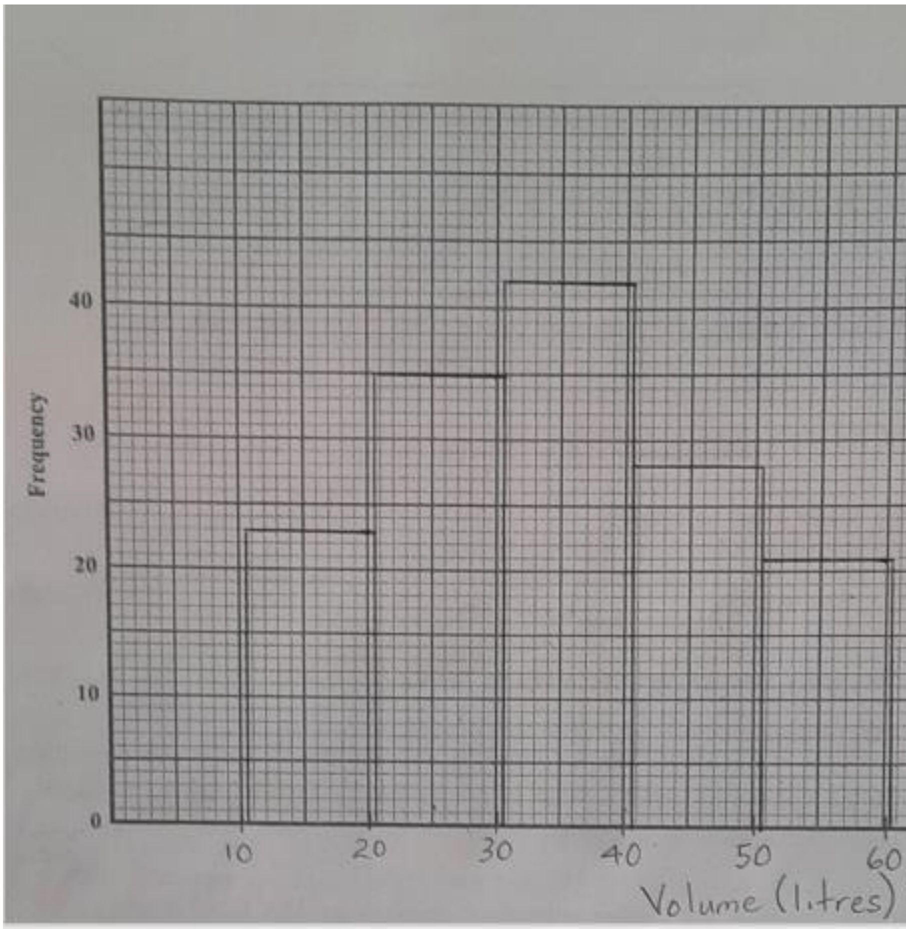
Now, $\frac{1}{2}\times 150 = 75$

So, the median is the 75^{th} term which lies in the interval $31-40$.

So, Byron's estimate is INCORRECT.

(e) Frequency distribution table

Volume (litres)	Frequency
$11-20$	24
$21-30$	$59-24=35$
$31-40$	$101-59=42$
$41-50$	$129-101=28$
$51-60$	$150-129=21$



(3 marks)

TRY THIS (May 2011)

15.

The table below shows the distribution of the masses of 100 packages.

Mass (kg)	No. of Packages	Cumulative Frequency
1 – 10	12	12
11 – 20	28	40
21 – 30	30	
31 – 40	22	
41 – 50	8	

(a) Copy and complete the table to show the cumulative frequency for the distribution. **(2 marks)**

(b) **Using a scale of 2 cm to represent 10 kg on the *x*-axis** and **1 cm to represent 10 packages on the *y*-axis**, draw the cumulative frequency curve for the data. **(5 marks)**

(c) Estimate from the graph

(i) the median mass of the packages **(2 marks)**

(ii) the probability that a package, chosen at random, has a mass which is LESS than 35 kg. **(3 marks)**

<u>Example (May 2021)</u>

(a) One hundred students were surveyed on the amount of money they spent on data for their cellphones during a week. The table below shows the results as well as the midpoint for each class interval.

Amount Spent ($)	**Number of Students (f)**	**Midpoint ($) ($x$)**
$50 < x \leq 60$	7	55
$60 < x \leq 70$	11	65
$70 < x \leq 80$	31	75
$80 < x \leq 90$	29	85
$90 < x \leq 100$	22	95

Using the table,

(i) a) determine the modal class of the amount of money spent (1 mark)

b) calculate an estimate of the mean amount of money spent, giving your answer correct to 2 decimal places (2 marks)

(a) (ii) Damion reports that the median amount spent is $84. Briefly explain why Damion's report could be correct (1 mark)

(b) The two-way/contingency table below gives information on the mode of transportation to school for 100 students.

	Walk	**Cycle**	**Drive**	**Total**
Boy	15		14	48
Girl		18	26	
Total	23		40	100

(i) Complete the table by inserting the missing values (2 marks)

(ii) A student is selected at random. What is the probability that he/she was being driven to school on that day? (1 mark)

(iii) One of the girls is selected at random. What is the probability that she did NOT cycle to school? (2 marks)

Solution

(a) (i) a) The highest frequency is 31, which corresponds to the interval

$70 < x \leq 80$. So, the modal class is $70 < x \leq 80$.

(i) b)

Amount spent $	Number of students (f)	Midpoint ($) x	fx
$50 < x \leq 60$	7	55	385
$60 < x \leq 70$	11	65	715
$70 < x \leq 80$	31	75	2325
$80 < x \leq 90$	29	85	2465
$90 < x \leq 100$	22	95	2090
	$\sum f = 100$		$\sum fx = 7980$

Now, mean $\bar{x} = \frac{\sum fx}{\sum f} = \frac{7980}{100}$

$= 79 \cdot 80$

So, the mean amount of money spent is $\$79 \cdot 80$

(ii)

Amount spent $	Cumulative frequency
$50 < x \leq 60$	7
$60 < x \leq 70$	18
$70 < x \leq 80$	49
$80 < x \leq 90$ ←	78 ← 50^{th} term
$90 < x \leq 100$	100

Total frequency, $n = 100$

Now, the median is the $\frac{1}{2}n^{\text{th}}$ term.

Now, $\frac{1}{2}\times 100 = 50$

So, the median is the 50^{th} term, which lies in the interval $80 < x \le 90$.

So, Damion's report could be correct.

(b) (i)

	Walk	Cycle	Drive	Total
Boy	15	19	14	48
Girl	8	18	26	52
Total	23	37	40	100

(ii) Number of students driven to school $= 40$

Probability that a student was driven to school $= \frac{40}{100}$

$= 0\cdot 4$

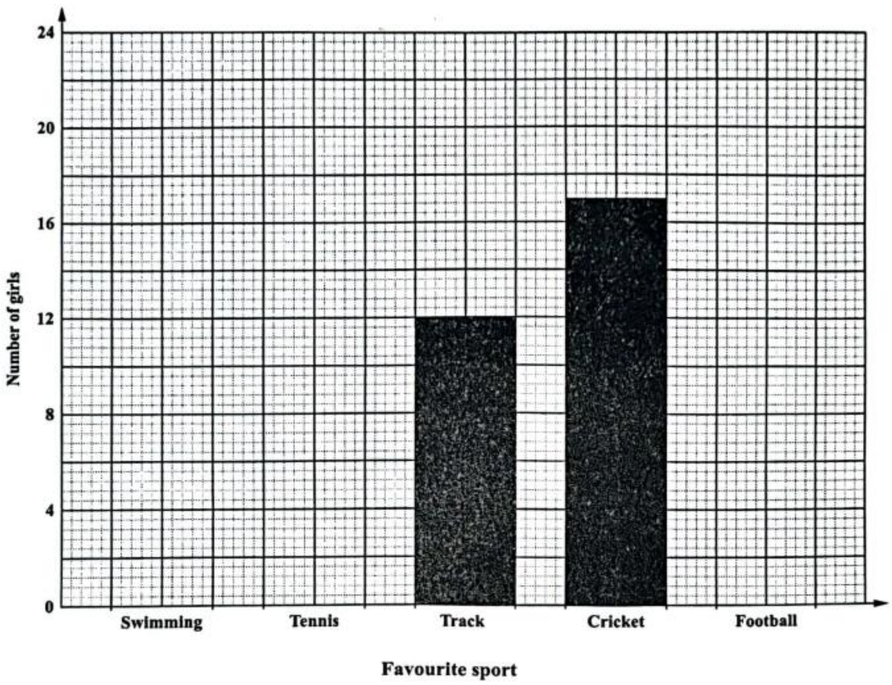
(iii) Number of girls who did NOT cycle to school $= 8 + 26 = 34$

Probability that a girl did not cycle to school $= \frac{34}{100}$

$= 0\cdot 34$

TRY THIS (May 2023)

16. Each of 75 girls recorded the name of her favourite sport. The number of girls who chose track and cricket are shown on the bar chart below.



(a) How many MORE girls chose cricket than track as their favourite sport? (1 mark)

(b) Eleven girls recorded tennis as their favourite sport. For the remaining girls, the number who chose swimming compared to the number who chose football was in the ratio 2:3

Use this information to complete the bar chart above. (3 marks)

(c) Determine the modal sport (1 mark)

(d) One of the girls is selected at random. What is the probability that she chose NEITHER track NOR cricket as her favourite sport? (2 marks)

(e) The information on the favourite sport of the 75 girls is to be shown on a pie chart. Calculate the sector angle for football. (2 marks)

Methods of data representation

Pie charts

A pie chart is a circular diagram which is used to represent statistical information.
The circle is divided into sectors of varying angles or areas.
Each sector (or area) is directly proportional to the magnitude of the information that it is representing.
A pie chart shows the size of each sector in relation to the entire information.

Example

Sixty students were asked to state their favourite subject. The information is in the table below:

Subject	English	Mathematics	History	Geography	French	Spanish
No. of students	10	16	12	8	8	6

Sector representing number of students who liked English $= \frac{10}{\cancel{60}_{1}} \times \frac{\cancel{360}^{6^\circ}}{1} = 60^\circ$

Sector representing number of students who liked Mathematics $= \frac{16}{\cancel{60}_{1}} \times \frac{\cancel{360}^{6^\circ}}{1} = 96^\circ$

Sector representing number of students who liked History $= \frac{12}{\cancel{60}_{1}} \times \frac{\cancel{360}^{6^\circ}}{1} = 72^\circ$

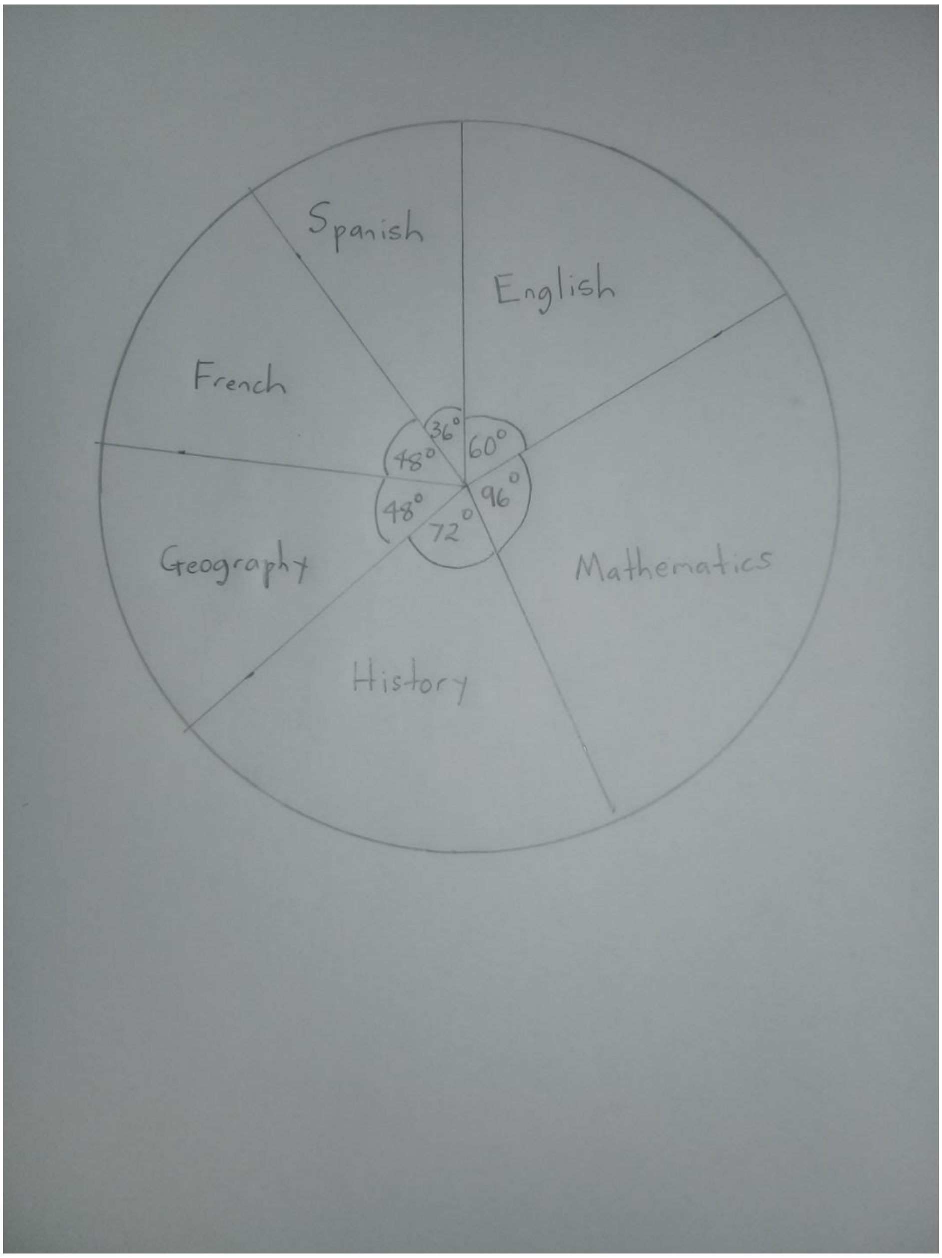
Sector representing number of students who liked Geography $= \frac{8}{\cancel{60}_{1}} \times \frac{\cancel{360}^{6^\circ}}{1} = 48^\circ$

Sector representing number of students who liked French $= \frac{8}{\cancel{60}_{1}} \times \frac{\cancel{360}^{6^\circ}}{1} = 48^\circ$

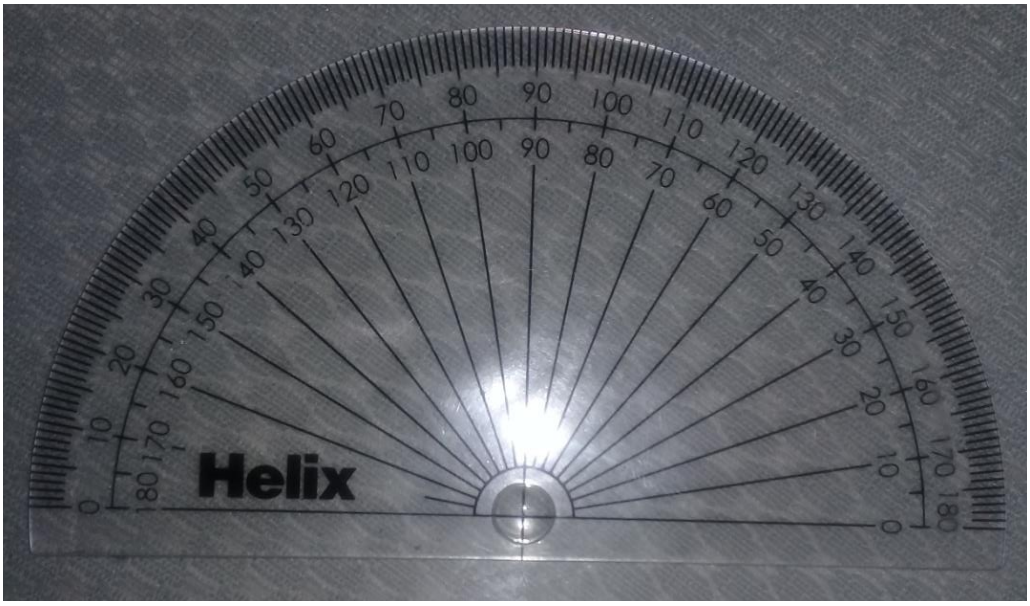
Sector representing number of students who liked Spanish $= \frac{6}{\cancel{60}_{1}} \times \frac{\cancel{360}^{6^\circ}}{1} = 36^\circ$

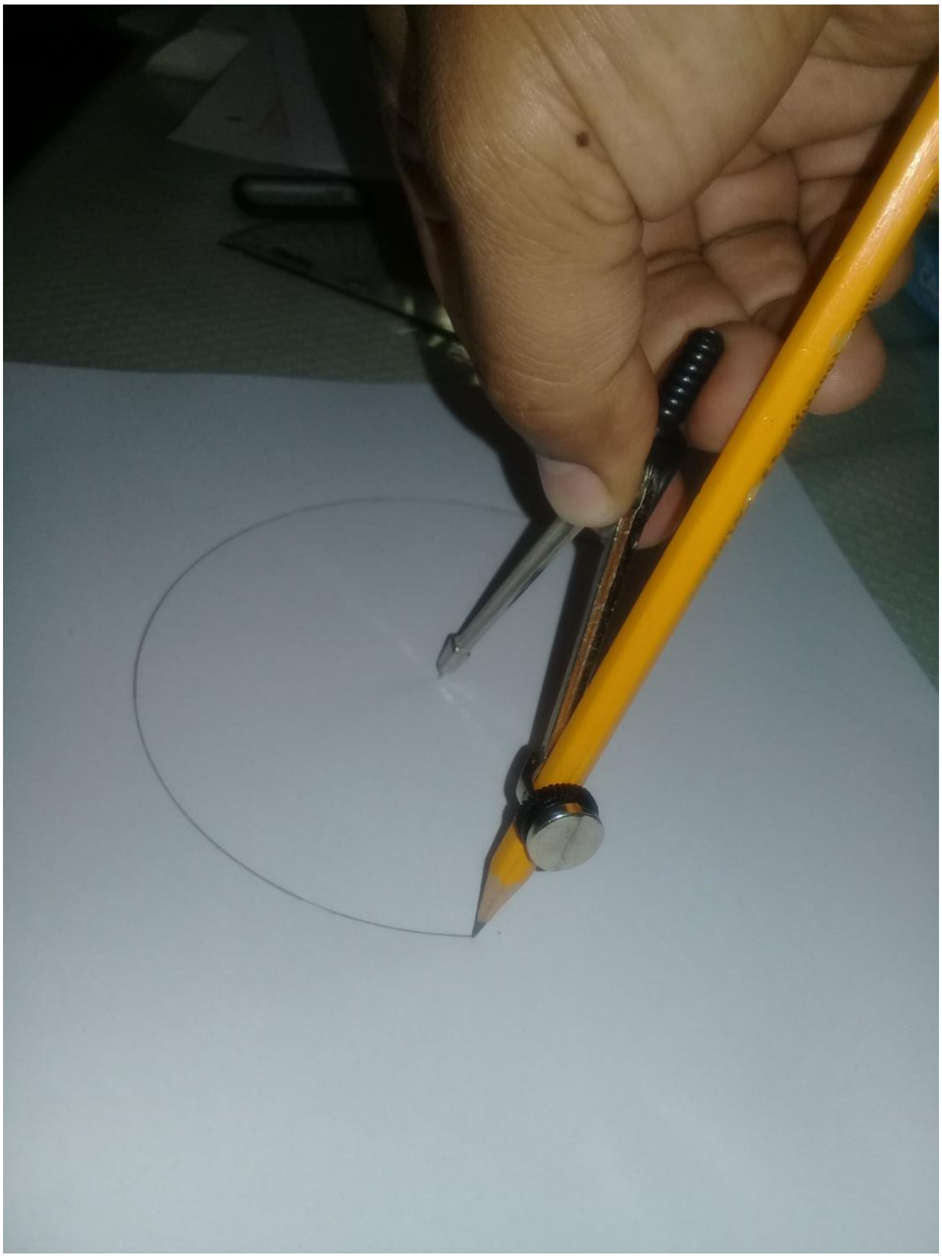
Note: $60^\circ + 96^\circ + 72^\circ + 48^\circ + 48^\circ + 36^\circ = 360^\circ$

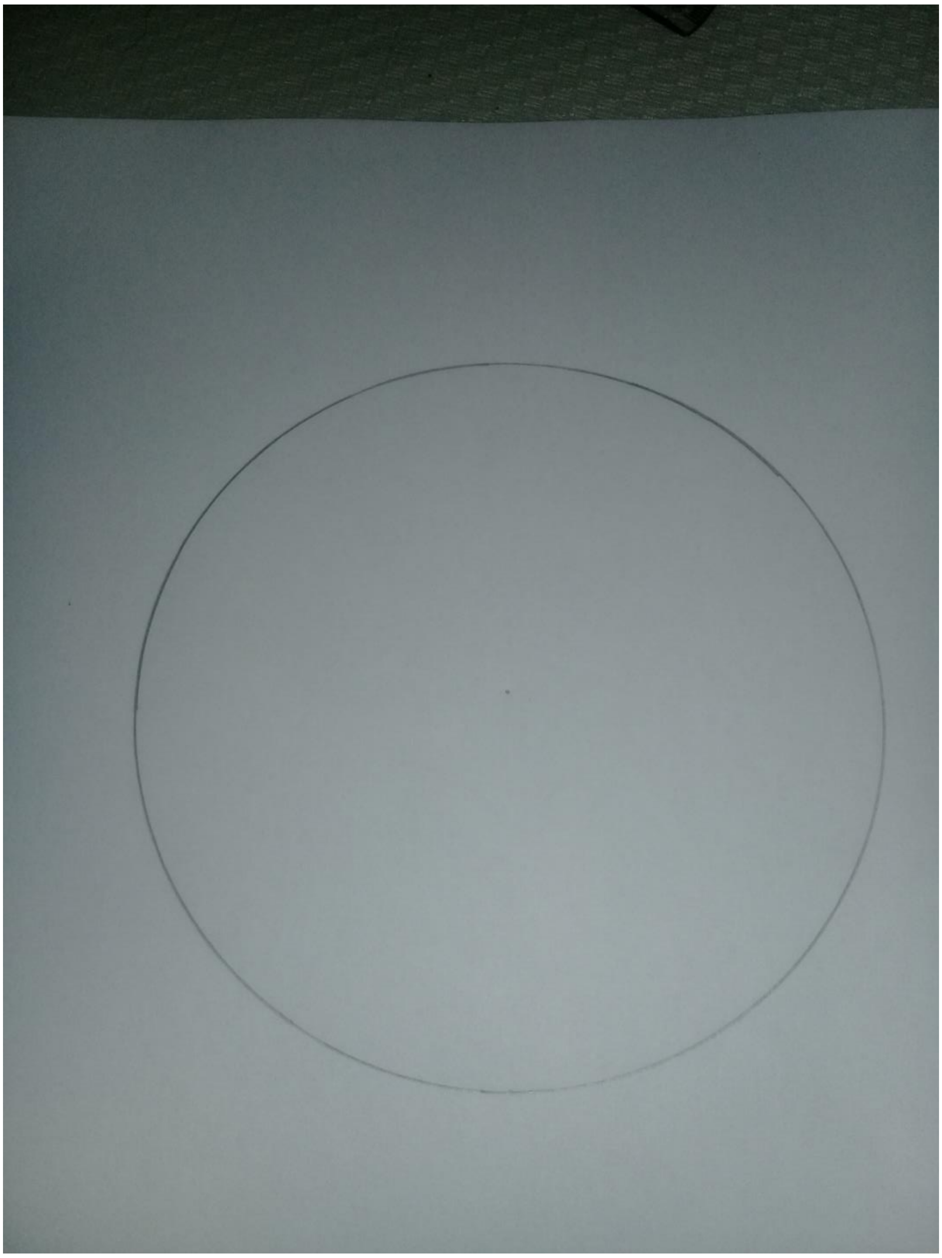
A pie chart representing the information above resembles:

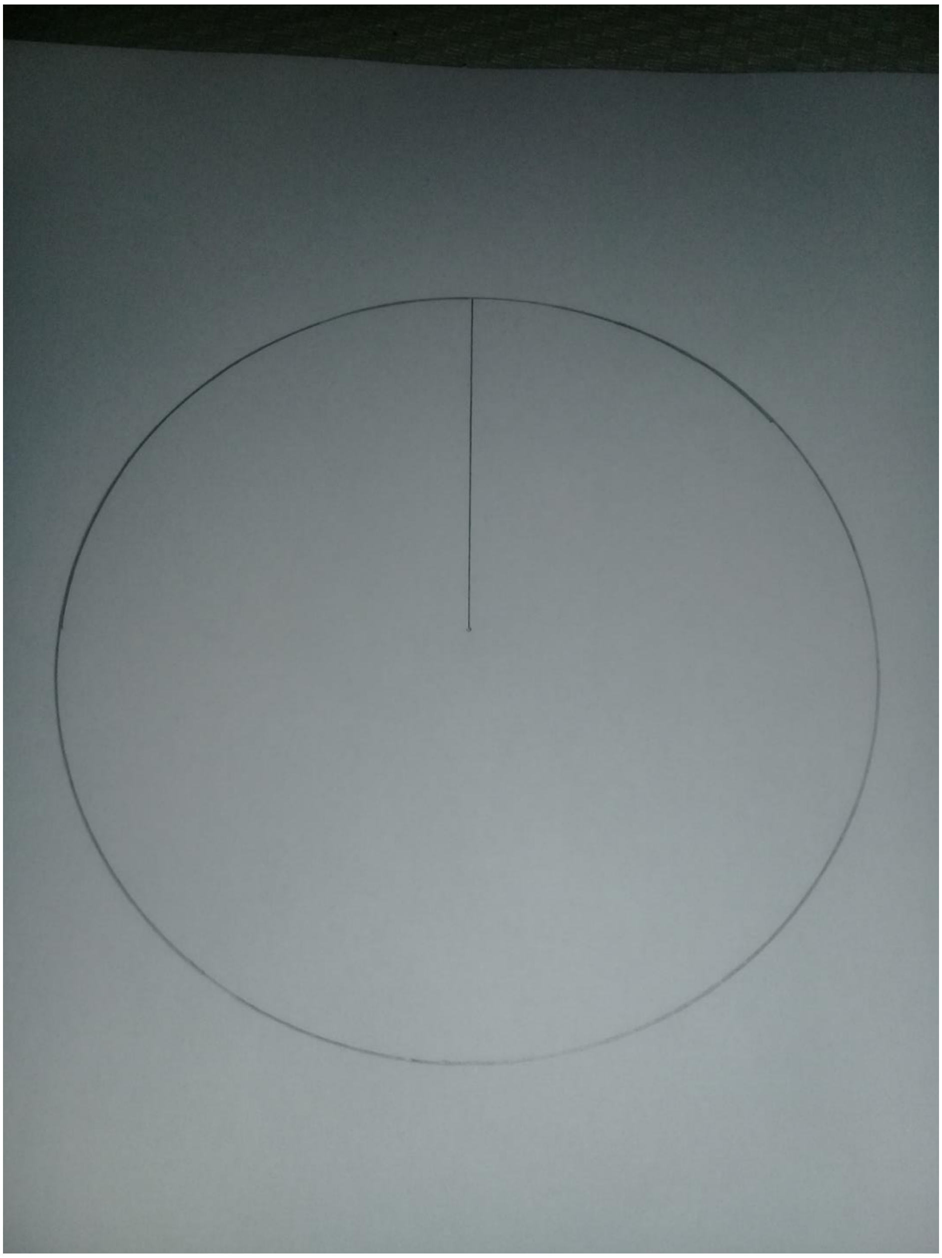

Spanish
English
French
36°
60°
48°
48°
96°
72°
Geography
Mathematics
History

PICTURES SHOWING THE CONSTRUCTION OF PIE CHART ABOVE

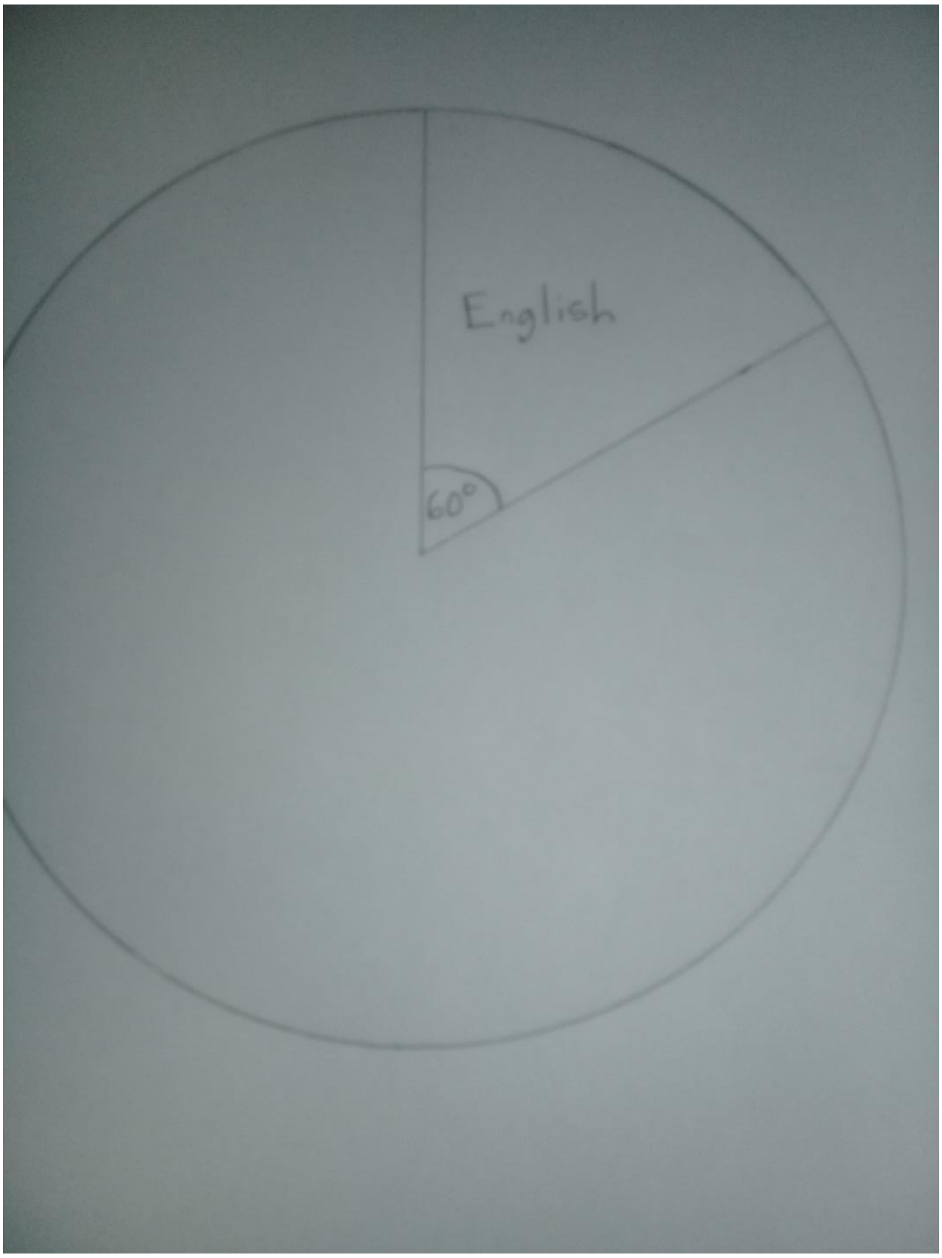


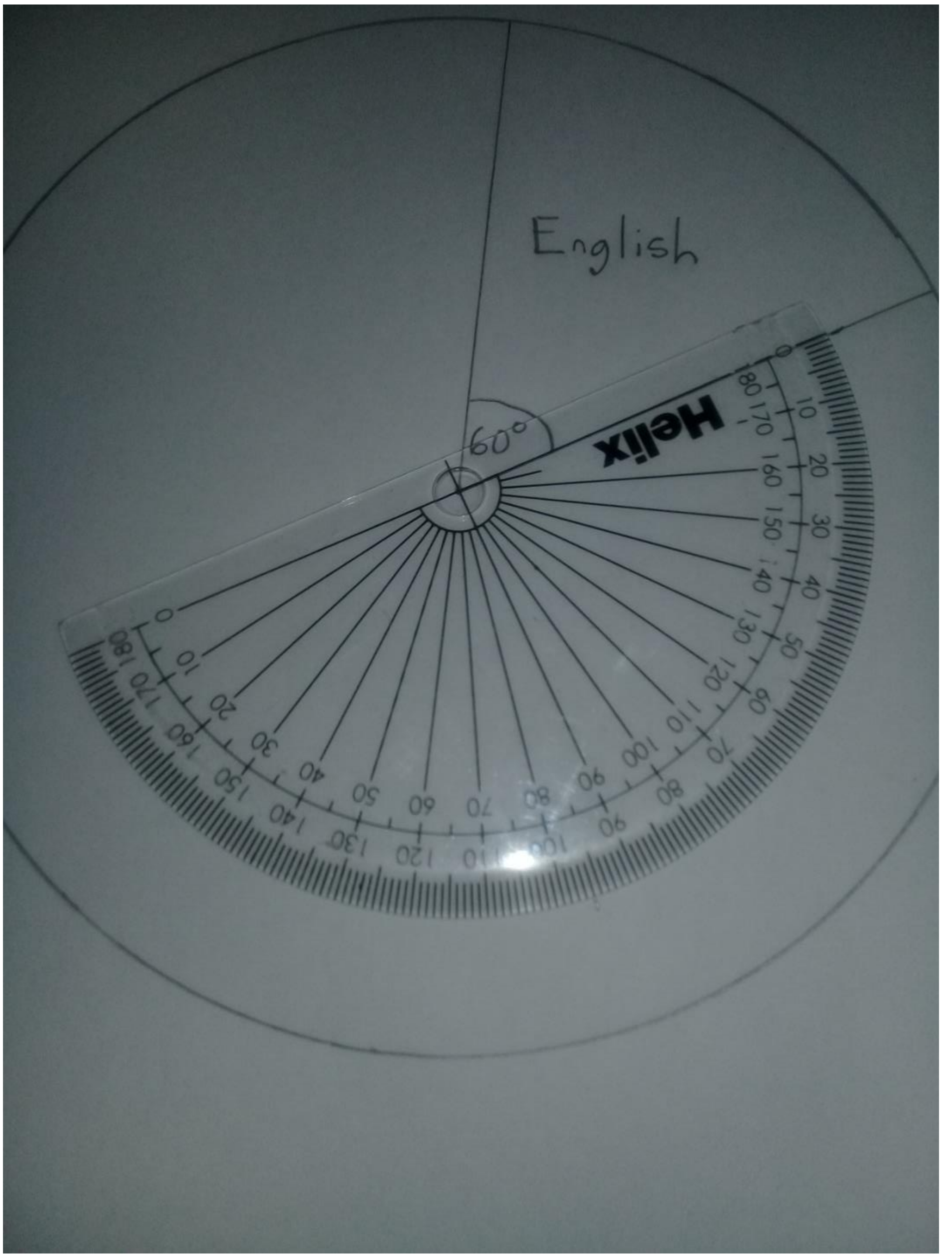


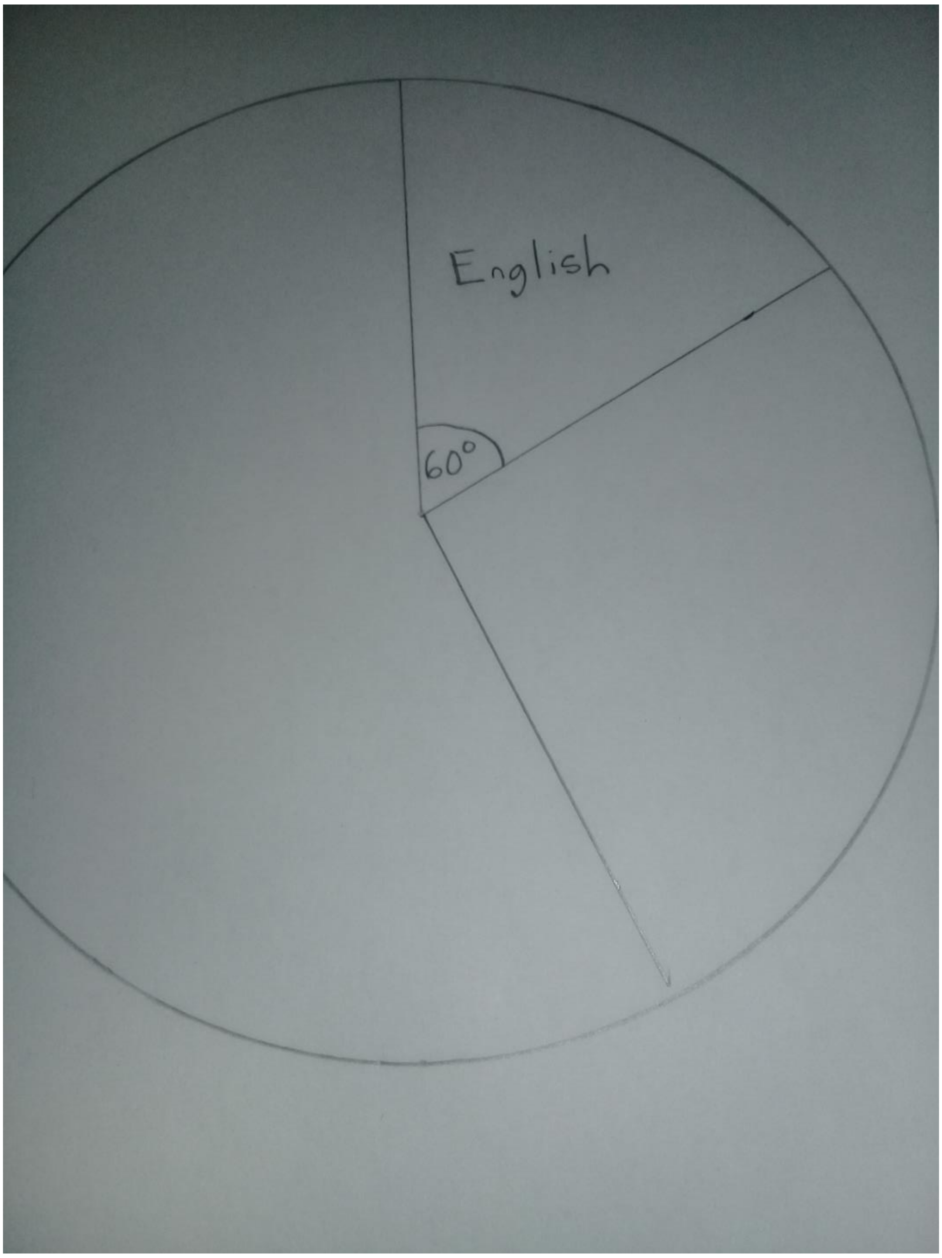


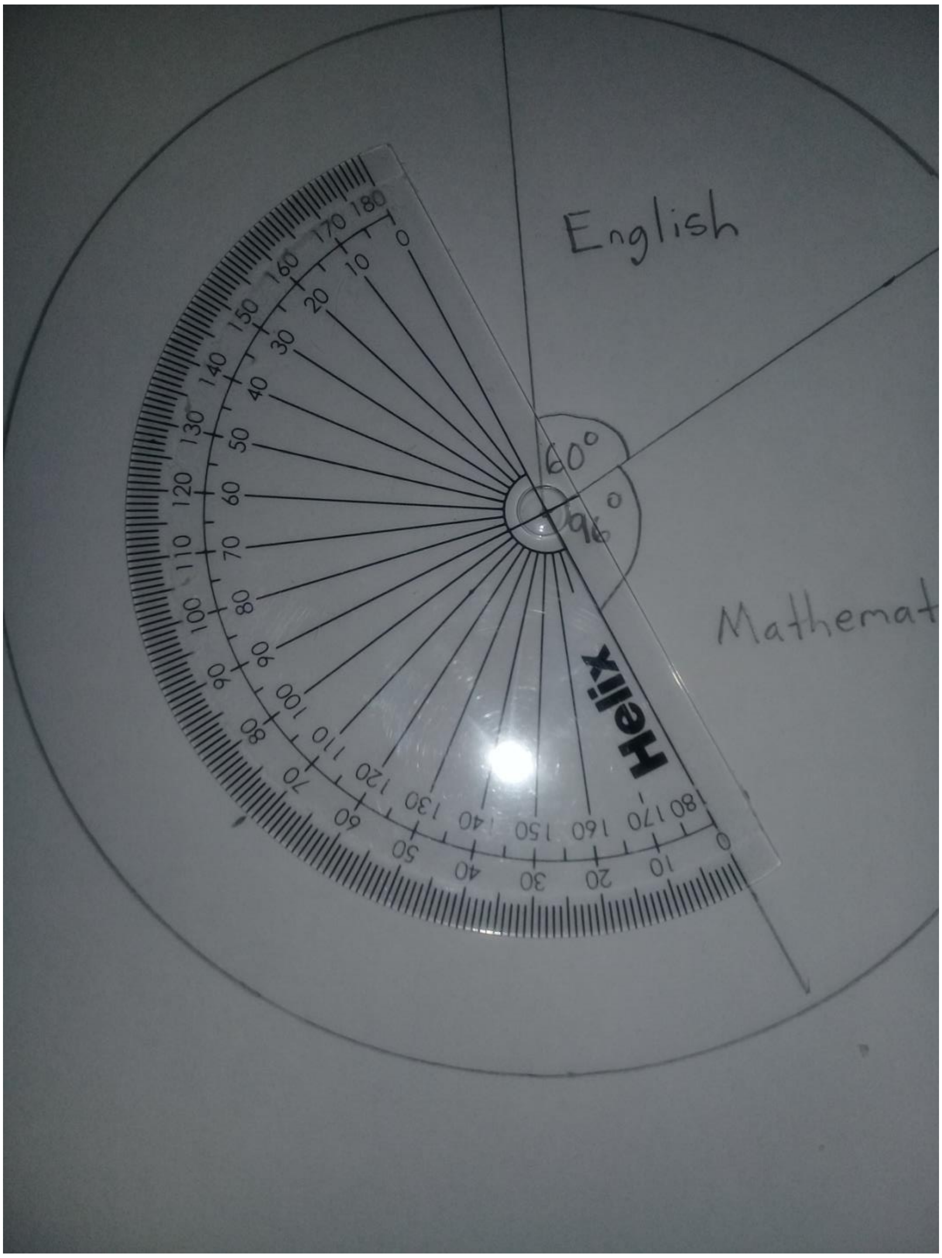


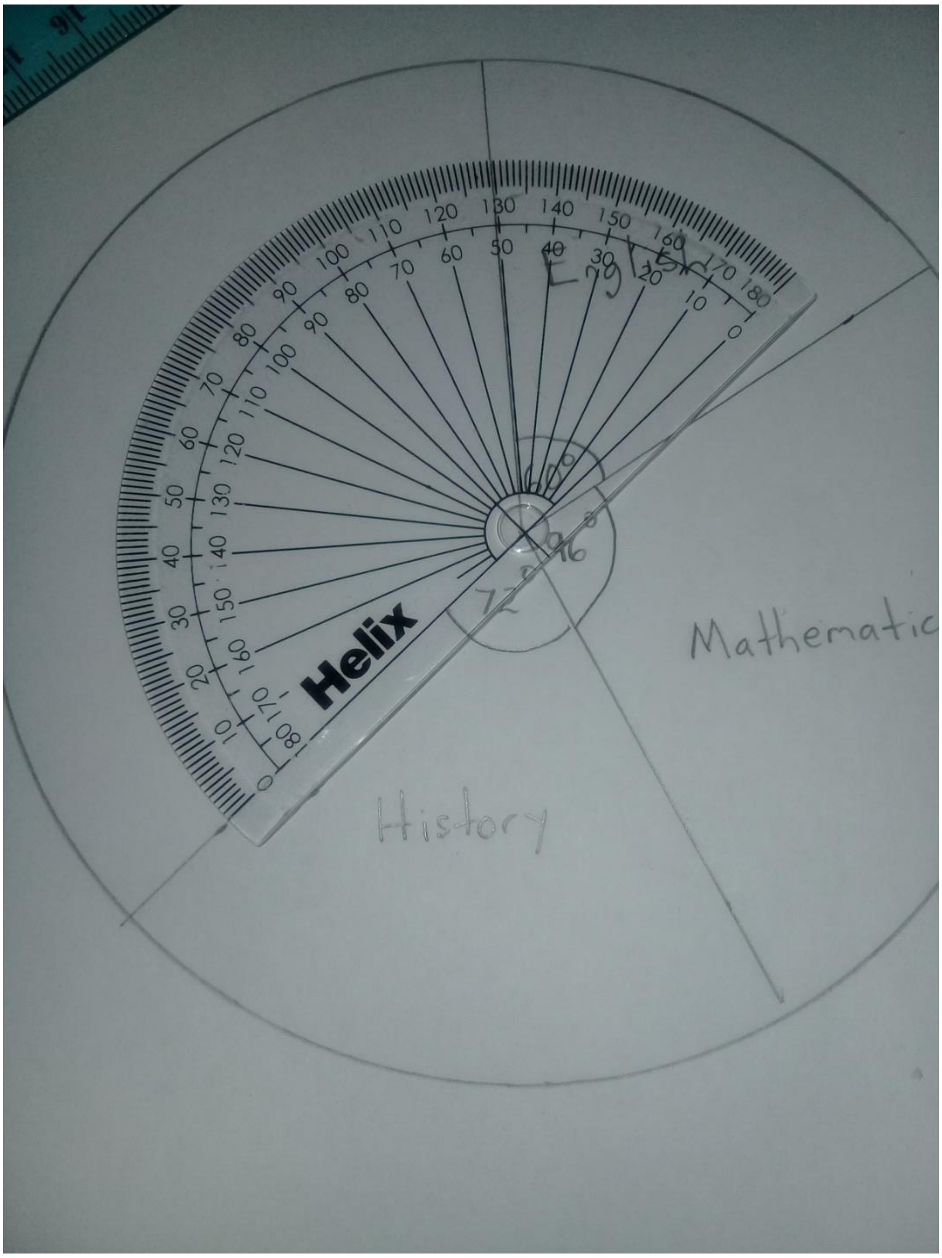

Helix

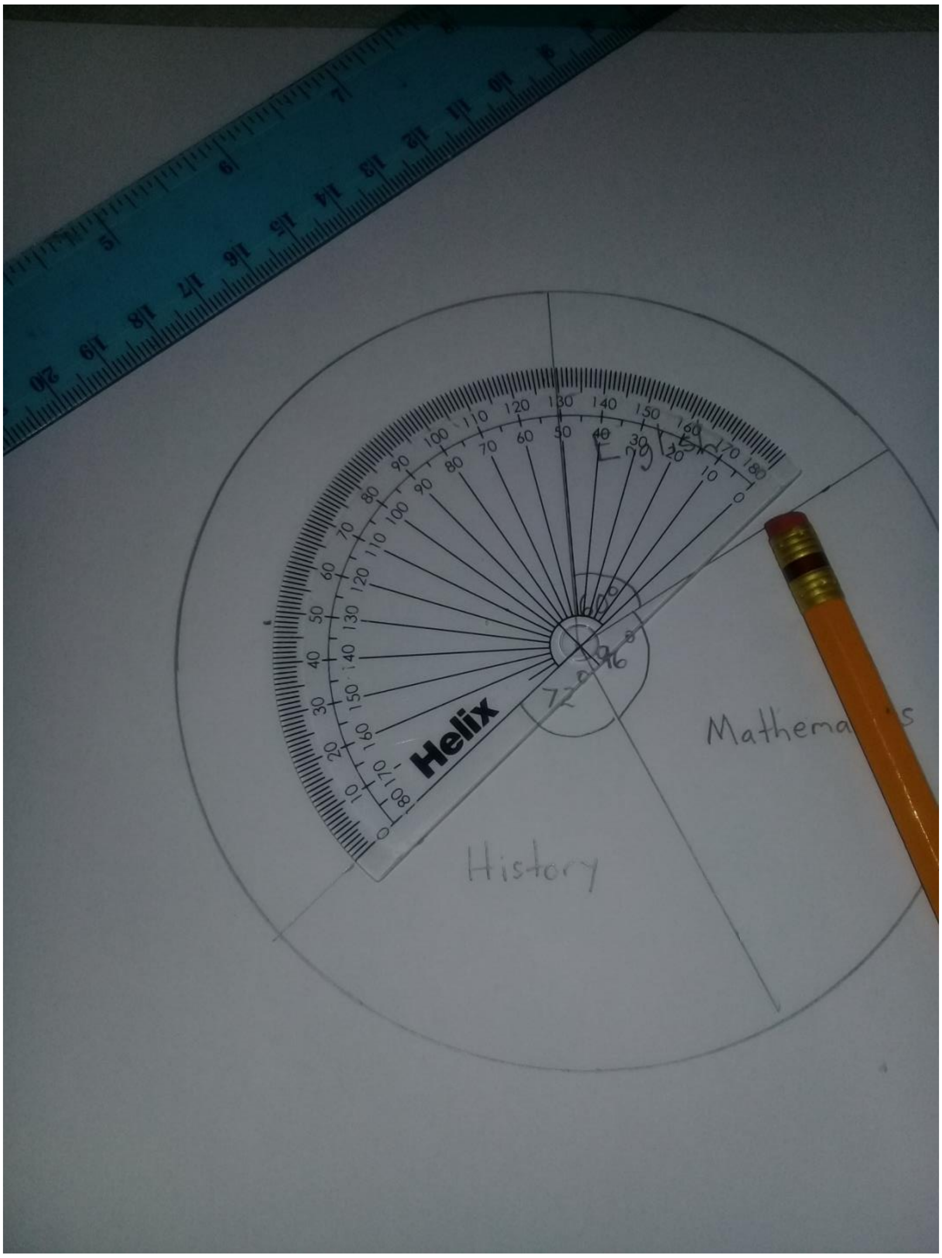

English
60°

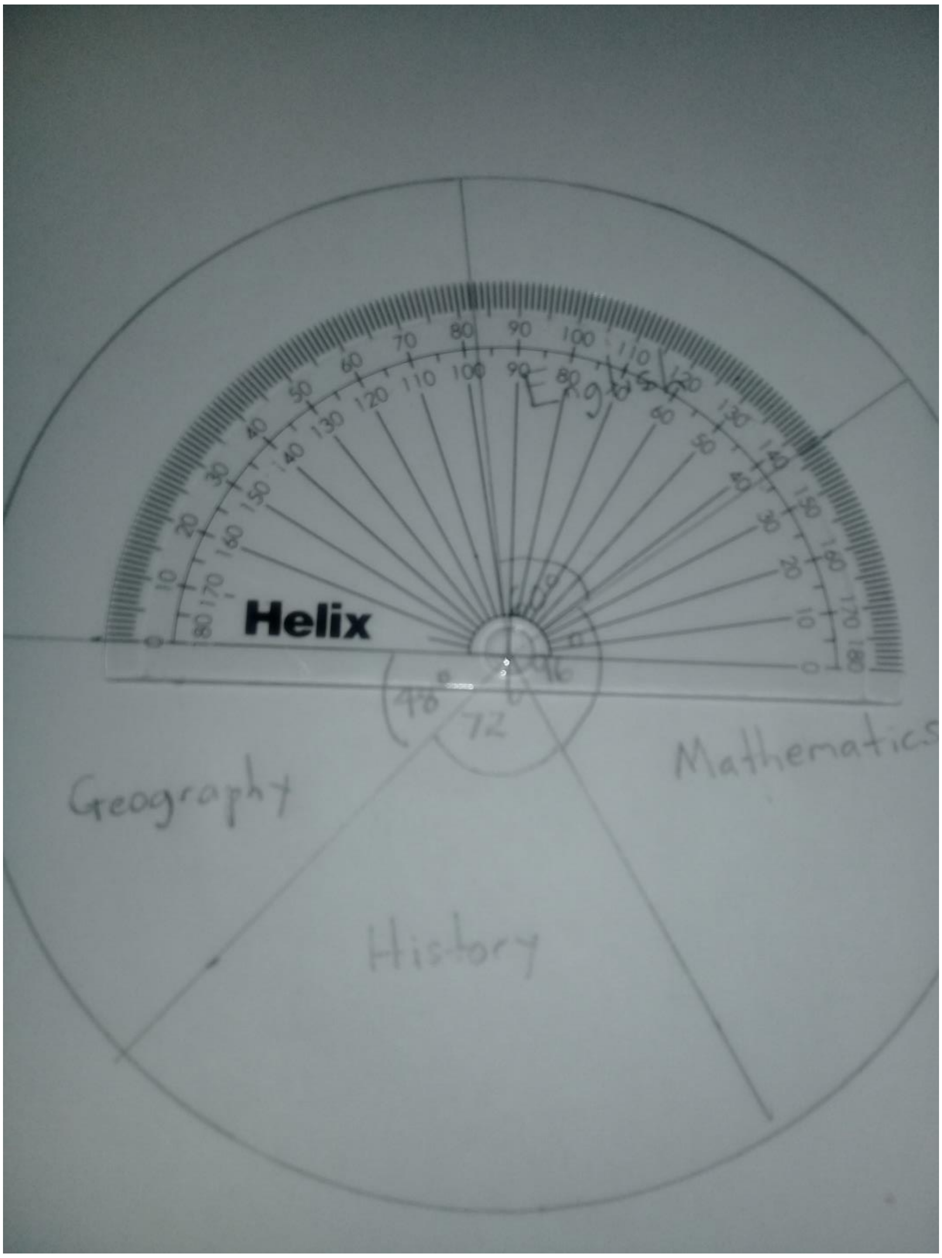

English
60°
Helix

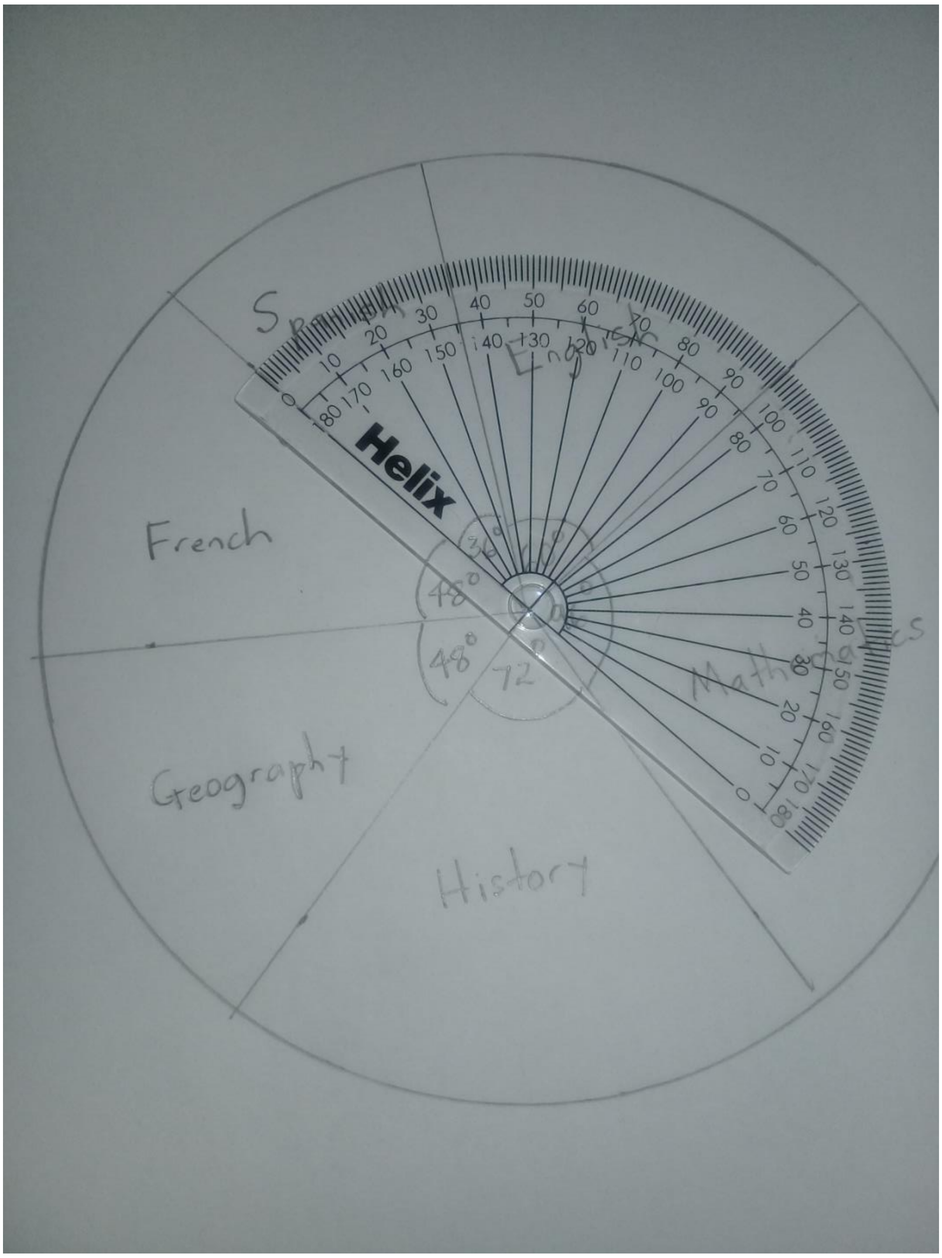

English
60°


English
60°
Mathemat
Helix


English
Mathematic
History


Helix
English
History


English
Mathematics
Geography
History
Helix
48°
72
96


Helix
French
Geography
History
48°
48°
72

Example (January 2021)

(a) Sixty students took an algebra test, whch comprised 15 multiple choice questions. The number of correct answers that each student obtained is recorded in the table below.

Number of Correct Answers	Number of Students
8	6
9	14
10	2
11	6
12	2
13	11
14	9
15	10

Using the table, determine

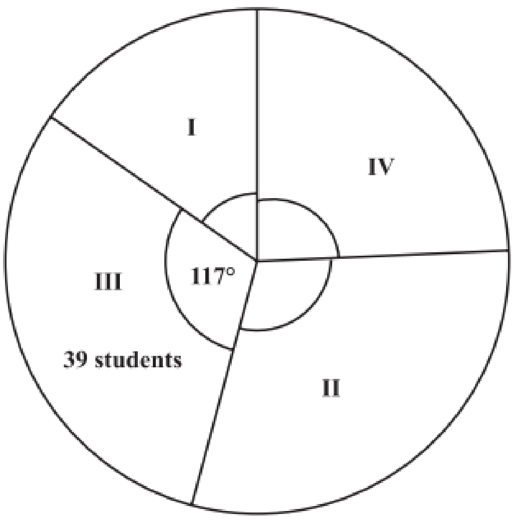
(i) the number of students who had exactly 13 correct answers (1 mark)

(ii) the modal number of correct answers (1 mark)

(iii) the median number of correct answers (1 mark)

(iv) the probability that a student chosen at random had at least 12 correct answers (1 mark)

(b) A group of students wrote a Physics examination. Each of the students achieved a Grade I, II, III or IV. The pie chart below shows the result.



Thirty nine students achieved a Grade III.

(i) Deterine the TOTAL number of students who wrote the examination (2 marks)

(ii) The ratio of the number of students who achieved a Grade I, II or IV is 2:4:3. A student passed the examination if he/she achieved a Grade I, II or III.
How many students passed the examination? (2 marks)

(iii) Determine the value of the angle for the sector representing Grade I in the pie chart. (1 mark)

<u>Solution</u>

(a) (i) Number of students who had exactly 13 correct answers is 11.

(ii) The highest frequency is 14, which corresponds to 9 correct answers. So, the modal number of correct answers is 9.

(iii)

Number of correct answers	Cumulative frequency	
8	6	
9	20	
10	22	
11	28	
12 ←	← 30 ←	30^{th} term
13 ←	← 41 ←	31^{st} term
14	50	
15	60	

Now, the median is the $\frac{1}{2}(n+1)$th rank

Here, $n = 60$

Now, $\frac{1}{2}(n+1) = \frac{1}{2}(60+1) = \frac{1}{2}(61) = 30.5$

So, the median is the 30.5^{th} term

Now, $\frac{12+13}{2} = 12\cdot 5$

$\therefore$ the median number of correct answers is $12\cdot 5$

(iv) Number of students who obtained at least 12 correct answers

$= 2+11+9+10$

$= 32$

Probability that student got at least 12 correct answers $= \frac{32}{60}$

$= \frac{8}{15}$

(b) (i) Let x represent the TOTAL number of students who wrote the examination.

Now, $117^\circ = 39$ students

So, $360^\circ = x$ students

Now, $x \times 117^\circ = 39 \times 360^\circ$

Now, $x = \frac{39 \times 360^\circ}{117^\circ}$

So, $x = 120$

$\therefore$ a TOTAL of 120 students wrote the examination.

(b) (ii) Since 120 students wrote the examination and 39 students got Grade III, then $120 - 39 = 81$ students got grades I, II and IV.

Number of students who got Grade IV $= \frac{3}{9} \times 81$

$= \frac{1}{3} \times 81$

$= 27$

Now, $120 - 27 = 93$

$\therefore$ 93 students passed the examination

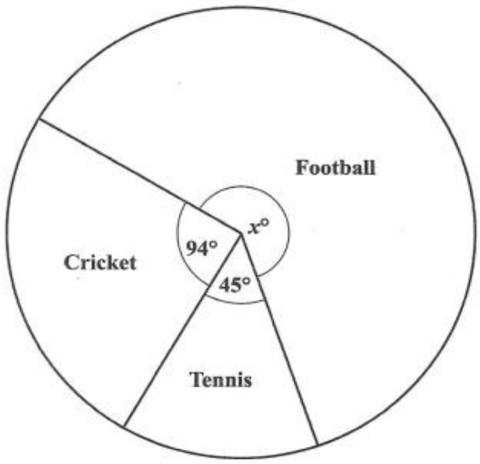
(iii) $360^\circ - 117^\circ = 243^\circ$

Now, $\frac{2}{9} \times 243^\circ = 54^\circ$

$\therefore$ the angle for the sector representing Grade I in the pie chart is 54°

TRY THIS (May 2018)

17. Students in a group were asked to name their favourite sport. Their responses are shown on the pie chart below.

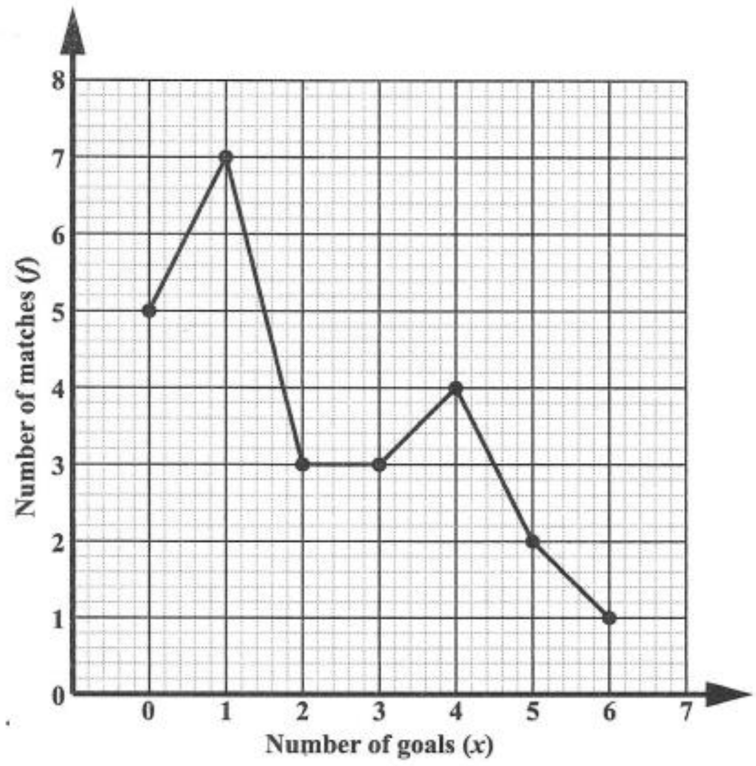


(i) Calculate the value of x (1 mark)

(ii) What percentage of the students chose cricket? (1 mark)

(iii) Given that 40 students chose tennis, calculate the TOTAL number of students in the group. (2 marks)

(b) The diagram below shows a frequency polygon of the number of goals scored by a football team in 25 matches.



(i) Complete the following table using the information in the diagram.

Number of matches (*f*)	5	7		3	4		1
Number of goals scored (*x*)	0	1	2	3	4	5	6

(1 mark)

(ii) What is the modal number of goals scored by the team? (1 mark)

(iii) Determine the median number of goals scored by the team (1 mark)

(iv) Determine the mean number of goals scored by the team (2 mark)

END OF MARATHON SESSION

ALL THE BEST ON MONDAY MAY 13

MARATHON BOOK CREATED BY **MR. RICARDO BARKER**

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